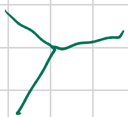
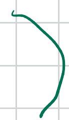
Pre-class Warm-up!!!

Divide the following letters into homeomorphism classes:

a b c d e f g h l n p q r

How many of these letters are in the same class as

a? With thickness 6 (abdepq) Is  ≅  ?

c?

h? 6 (cfhlnr) (no thickness)

Answers:

No thickness: a abdpq

c cl

h hrn

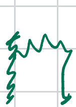
1

2

3

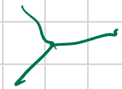
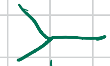
4

5

 ≅ 

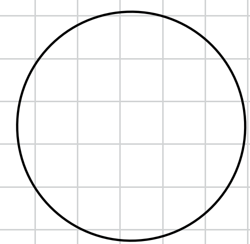
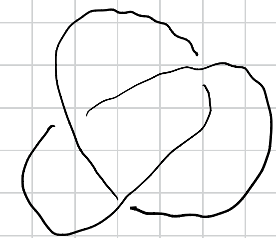
How to show spaces are not homeomorphic:

Remove a finite number of points and see what you get

 ≇ because it is possible to remove a point from  so as to get 3 pieces, and with | this is not possible

e ≇ a We can remove 2 points from a and get 4 pieces. This is not possible with e

Knots:

 ≅ 

as sets with the induced topology

Their embeddings in $\mathbb{R}^3$ are distinct.

Exercises 4.5

(a) If $X \subsetneq Y \subsetneq Z$ and Y has the subspace topology induced from Z, then the topology induced on X as a subset of Y is the same as the topology on X induced as a subset of Z.

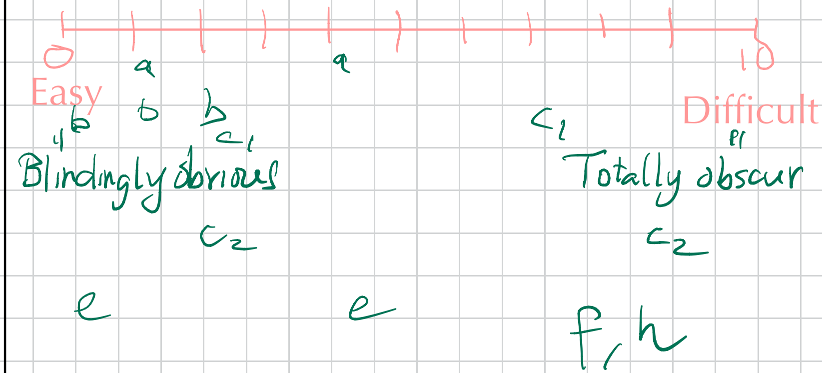
(b) A subspace of a metrizable space is metrizable.

(c) If S is a subspace of X then the inclusion map is continuous; furthermore S has the weakest topology so that the inclusion map is continuous.

(e) If A is a subset of S, the closure of A in S is contained in the closure of A in X, and they might not be equal.

(f) and (h) $(0,1)$, $(0, \text{infinity})$ and $\mathbb{R}$ are homeomorphic.

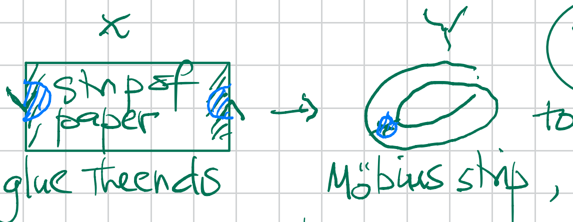
Where on the difficulty scale do you put these?

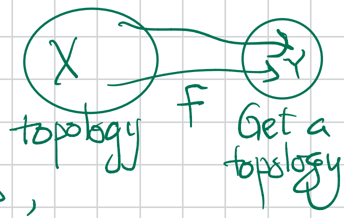


Chapter 5: Quotient topology (and groups acting on spaces)

Definition 5.1. Let f : X -> Y be a surjective mapping from a topological space X to a set Y. The quotient topology on Y (with respect to f) is the family

$\mathcal{U}_f$ = { U | f^{-1}(U) is open in X }





open set in X = f^{-1}(set shown in Y)

We put a topology on the Möbius strip.

Proposition. (a) If Y has the quotient topology then f is continuous.
(b) The quotient topology is the smallest topology on Y so that f is continuous.

Proof (a) We check $\forall$ open $U \subseteq Y$, $f^{-1}(U)$ is open to verify that f is continuous. This holds from the definition of open sets in Y.

b). in any topology on Y for which f is continuous, $\forall$ open $U \subseteq Y$, $f^{-1}(U)$ must be open, so this topology must contain the quotient topology. $\square$

True or false? Let S be a subset of a topological space X.

(i) The induced topology on S is the largest topology on S so that the inclusion map g : S -> X is continuous. T/F?

(ii) The induced topology on S is the smallest topology on S so that the inclusion map g : S -> X is continuous. T/F?

Examples for the quotient topology

Example: projective space.

Questions:

1. Have you ever encountered a theorem in projective geometry before? Yes / No
2. How many ways have you seen before of constructing a projective space, such as the real projective plane RP^2 ?

   a. 0
   b. 1
   c. 2
   d. 3
   e. >3