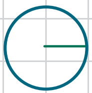
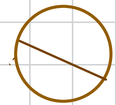
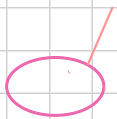
Pre-class Warm-up!!!

Are any of the following spaces homeomorphic?

A 

B 

C 

D 

a. No two of them are homeomorphic

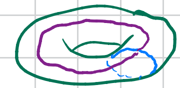
b. Precisely two are homeomorphic ✓ $A \cong C$

c. Two are homeomorphic and the other two are homeomorphic

d. A is homeomorphic, but B, C and D are not.

Another question: Let S^1 denote the unit circle in R^2 . Consider the torus T = S^1 x S^1 contained in R^2 x R^2 = R^4.

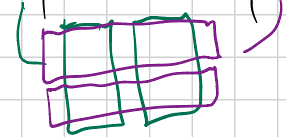
Do you think the complement of T in R^4 is connected?

$T =$  without the inside.

$$\mathbb{R}^2_a = S^1_a \sqcup K_a \qquad \mathbb{R}^2_b = S^1_a \sqcup K_b$$

$$\mathbb{R}^4 = S^1_a \times S^1_b \sqcup K_a \times \mathbb{R}^2_b \sqcup \mathbb{R}^2_a \times K_b$$

2 pieces 2 pieces



The four pieces overlap as shown, and each piece is connected.

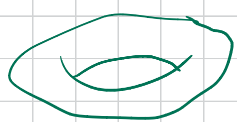
Compact surfaces

Surfaces are 2-dimensional manifolds (without boundary).
Examples:

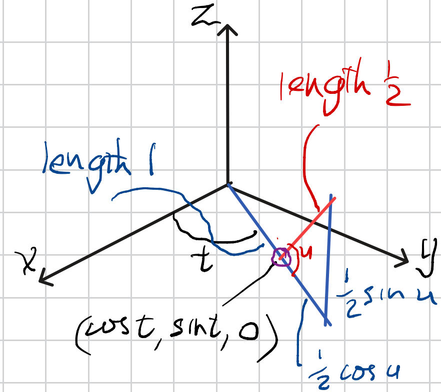
a. The torus $T = S^1 \times S^1 \subseteq \mathbb{R}^4$.

Proposition. Products of manifolds are manifolds.

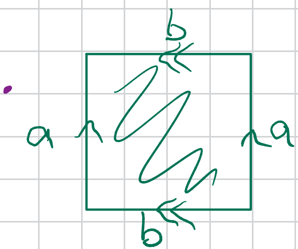
Proof. A product of Hausdorff spaces is Hausdorff. If $(x,y) \in M \times N$, $x \in U_x \subseteq M$, $y \in V_y \subseteq N$ are open nbds homeomorphic to open balls, then $(x,y) \in U_x \times U_y$ is an open nbd of $M \times N$, homeom to an open ball. $\square$

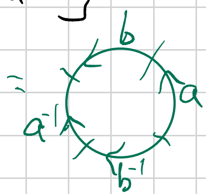
b. Also $T =$  $\subseteq \mathbb{R}^3$

Which description of the torus do you like best?
a. S^1 x S^1   b. The subset of R^3
c. The identification space a b a^{-1] b^{-1} ?



$$T = \left\{ \left(\left(1 + \tfrac{1}{2}\cos u\right)\cos t,\ \left(1 + \tfrac{1}{2}\cos u\right)\sin t,\ \tfrac{1}{2}\sin u \right) \;\middle|\; t, u \in \mathbb{R} \right\}$$

c. 

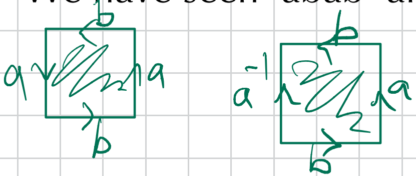
$= aba^{-1}b^{-1} =$ 

Convention: read round the circle counterclockwise, + arrows go counterclockwise.

Best. c 8 a 2 b 0

Identification spaces again

We have seen abab and aba^{-1}b.



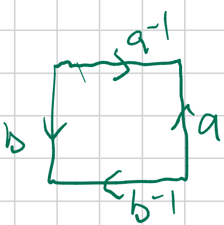
$= K$ the Klein bottle.

$\mathbb{RP}^2$ the projective plane

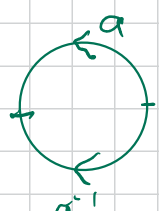
These are compact 2-manifolds = compact surfaces.
Is it obvious that the corners have neighborhoods homeomorphic to an open disk?

totally obscure

blindingly obvious.

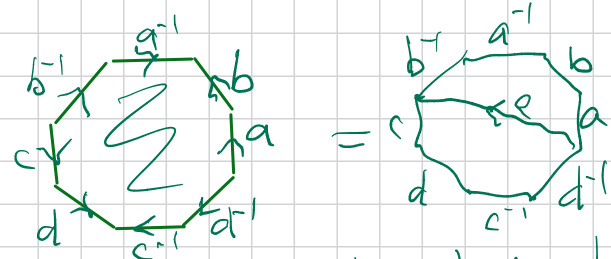


$aa^{-1}bb^{-1} \cong S^2 \cong$



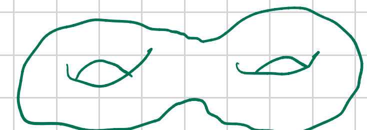
Note bb^{-1} cancels.

What about aba^{-1}b^{-1}cdc^{-1}d^{-1} ?



glue the boundaries of the holes together

together:



a double torus

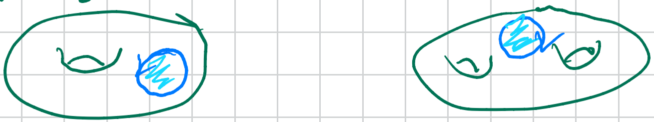
= 2-sphere with 2 handles.

Theorem. (See page 77 of the book) The only compact 1-manifold is S^1.

The connected sum of two surfaces

Page 79 of Kosniowski's book. We will not do things 'rigorously' in the sense of the book.

Given two surfaces M and N remove a small open disk from each and identify M – open disk and N – open disk along the remaining boundaries of the disks. We get the connected sum $M \# N$.



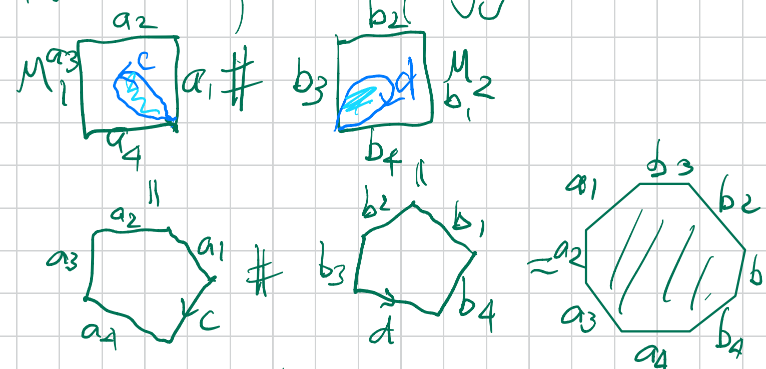
missing

identify the blue circles.

The double torus aba^{-1}b^{-1}cdc^{-1}d^{-1} is the connected sum of two tori: T # T.

We have seen by example that if we take the connected sum of

Two identification polygons



We get another polygon where the words describing the edges are juxtaposed.

$$a_1a_2a_3a_4 \# b_4b_1b_2b_3$$

$$= a_1a_2a_3a_4b_4b_1b_2b_3$$

$$\overset{?}{\cong} a_1a_2a_3a_4b_1b_2b_3b_4$$

Theorem. If M is a surface then S^2 # M is homeomorphic to M.