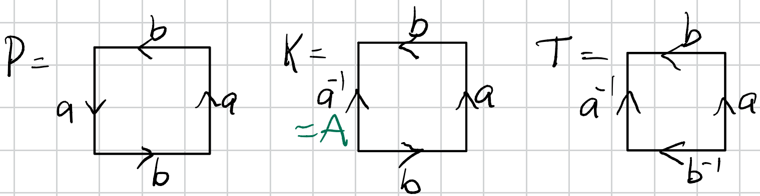
# Pre-class Warm-up!!

You recall:

the projective plane: RP^2 = P = abab

the Klein bottle: K = aba^{-1}b = abAb

the torus: T = aba^{-1}b^{-1} = abAB




Which of the following can you identify?

a. bAba ( A = a^{-1}) K

b. cc think c = ab P

c. abCcAB T

d. Aa $S^2$ Aa ≈ ACca

e. ABaB K ABaB = $(bAba)^{-1}$.

f. ~~abab~~ Better: abcabc P Rewrite bc = d

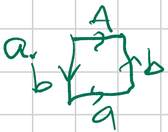
g. abABcdCD double torus.

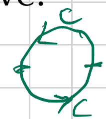
(i) P

(ii) K

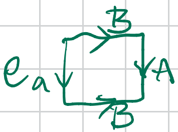
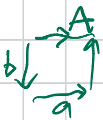
(iii) T

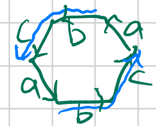
(iv) None of the above.

a.  rotation

b.  rewrite symbols

c.  cancellation

e  = inverse of , flip the paper

f  Rewrite symbols

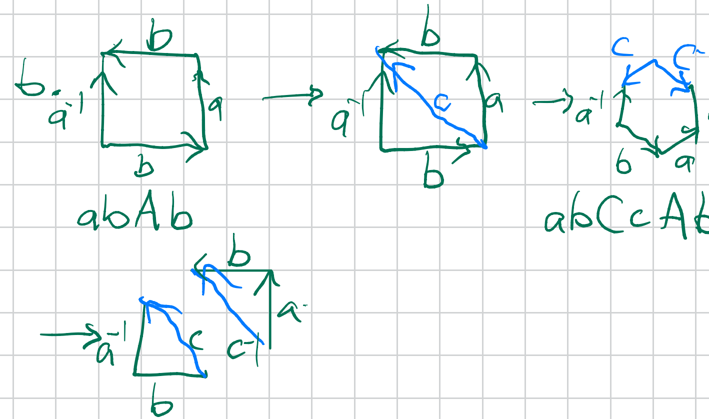
g two subwords with disjoints symbols: connected sum.

The double torus aba^{-1}b^{-1}cdc^{-1}d^{-1} is the connected sum of two tori: T # T.

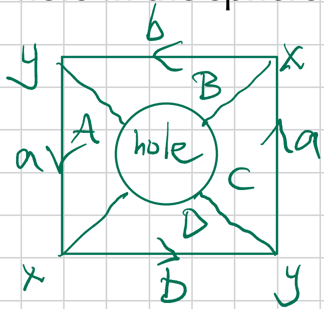
Rules for transformation of surface diagrams:

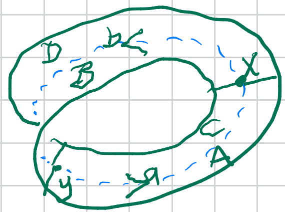
1. Each letter appears exactly twice, in the form y or y^{-1} = Y
2. Relabel the words letters (including a <-> a^{-1} = A).
3. Cyclically permute words: abc <-> bca.
4. Flip words: w = aBc <-> W = CbA.
5. (Un)cancel: yaAz <-> yz
6. (Un)split words: wCcx <-> wC and cx
7. (Un)write strings as single letters, such as: w ab x BA y <-> w c x C y
8. From a word w x where the letters in w are disjoint from those in x, we get a connected sum.

See Secs. 76-78 of Munkres.



A 2-sphere with a Möbius strip sewn onto a hole in the sphere is a projective plane P.



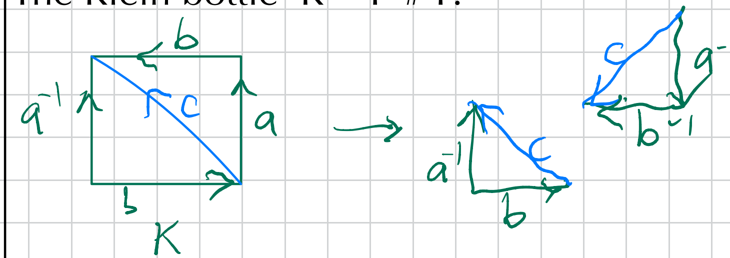




$\simeq P$

Or

= Putting a hole in a projective plane gives a Möbius strip.

Write P instead of RP^2 for the projective plane.
The Klein bottle K = P # P.



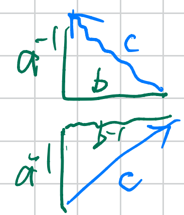

$abAb \to abCcAb \to abC\ cAb$

$\to bCa \quad bcA \to AcB \quad bcA$

$\to Ac\cancel{Bb}cA \to AccA$

$\to AAcc \to aacc \simeq aa \# cc$

$= P \# P.$



The relation T#P = P#P#P.

Rules for transformation of surface diagrams:

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Useful move $a y_1 a y_2 \to a y_1 c^{-1} c a y_2$

$\to c y_1^{-1} a^{-1} a y_2 c \to c y_1^{-1} y_2 c$

$\to c c y_1^{-1} y_2$.

Proof $P \# T = P \# P \# P$

$P \# T = c c a b a^{-1} b^{-1} = \underset{\text{rotate}}{c a b m^{-1}} \, \underset{\text{invert}}{m a^{-1} b^{-1} c}$

$\to a b m^{-1} c \, c^{-1} b a m^{-1} = a b m^{-1} b a m^{-1}$

$= \underbrace{a b n}_{y_1} b a \underbrace{n}_{y_2}$

$\to \underbrace{c c}_{P} \# \underbrace{b^{-1} n^{-1} b^{-1} n}_{K}$

## Main goal

Theorem 11.3. Every compact surface is homeomorphic to precisely one of the following:

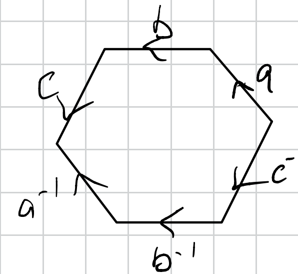
$S^2 \# \underbrace{T \# T \# \dots \# T}_{m \geq 0}$ or $\underbrace{P \# P \# \dots \# P}_{n \geq 1}$

There is a relation T#P = P#P#P.

Sections 76 and 77 of Munkres: Topology are more specific than Kosniowski's book.

Preliminary to that: simplicial complexes and triangulations.

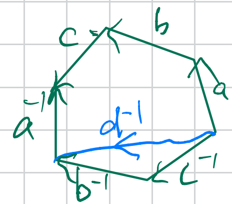
Activity: Determine which surface on the list corresponds to abcABC

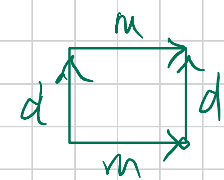



$abcdd^{-1}a^{-1}b^{-1}c$

$\to bcdd^{-1}aa^{-1}b^{-1}c \to bcdd^{-1}b^{-1}c$

$\to ?$




$abca^{-1}b^{-1}c^{-1} \to abca^{-1}d^{-1}db^{-1}c^{-1}$

$\to db^{-1}c^{-1} \sqcup ca^{-1}d^{-1}ab$

$\to db^{-1}a^{-1}d^{-1}ab$

$m = ab$

$\to dm^{-1}d^{-1}m$ torus T