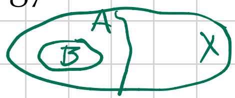
Pre-class Warm-up!!

Let A be a subset of a space X, and let A have the induced topology. Let B be a subset of A.
True or false?



a. If A is closed in X and B is closed in A, then B is closed in X. T F

Closed sets B have the form $A \cap C$, C closed in X. This B is closed in X.

b. If A is closed in X and B is open in A, then B is open in X. T F

c. If A $= (0,1) \subseteq \mathbb{R}$ is open in X and B $= (0, 1/2]$ is closed in A, then B is closed in X. T F

d. If A is open in X and B is open in A, then B is open in X. T F

Another question:

Let Δ be an abstract simplicial complex with vertices {a, b, c} and other simplices { {a,b}, {b,c}, {c,a} }.
What does the geometric realization of this simplicial complex look like?

a. a closed disk in R^2.
b. the closed interval [0,1]
c. a circle S^1
d. something else

What are open nbds of 0 in $[0,1] \subseteq \mathbb{R}$ (= A). They include $[0,\frac{1}{2})$

$[0, 1/2)$ is open in A, Not open in $\mathbb{R}$.

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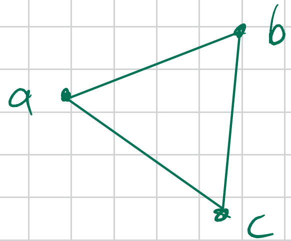
Geometric realization

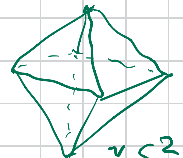


$\cong S^1$

Triangulations of manifolds

Definition. A manifold is triangulable <=> it is homeomorphic to a simplicial complex.

The simplicial complexes that arise for surfaces have the property that each edge is an edge of exactly 2 triangles (and the complex has dimension 2).



$\cong S^2$

Theorem. Every compact surface is triangulable by a finite simplicial complex.

I have never seen a proof of this.

How obvious is the following?

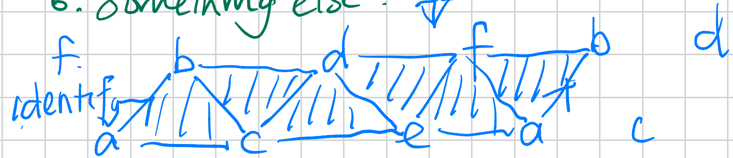
Theorem. Every compact triangulable surface can be made by taking a polygonal piece of paper and glueing up the edges in some way.

What if we have 53 triangles all meeting at a single vertex?

Do: Worksheet 12 qn 1(a).

2 simplices: abc, bcd, cde, def, efa, fab

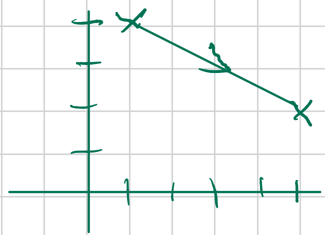
1. S^1
2. projective plane
3. cylinder
4. Möbius strip
5. S^2
6. Something else.



Chapter 12. Paths and path connected spaces.

Definition. A path in a topological space X is a continuous mapping $f : [0,1] \longrightarrow X$.

Example $f(t) = (5,2)t + (1,4)(1-t)$



The image of f is called a curve in X

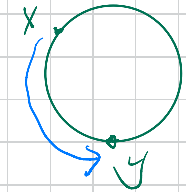
The path goes from $f(0)$ to $f(1)$

Example: This path goes from $(1,4)$ to $(5,2)$.

A space X is path connected $\Longleftrightarrow$ there is a path from x to y for all pairs of points $x, y \in X$.

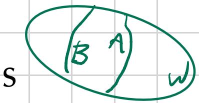
Example $\mathbb{R}^n$ is path connected.

So is S^n, but it's harder to describe the paths



Multiplication of paths and the inverse of a path

Glueing lemma for continuous functions:
Lemma 12.2.
Suppose the space W is the union of two closed subsets A and B and
f : A -> X, g : B -> X are two continuous functions that agree on A intersect B.
Then h : W -> X defined by



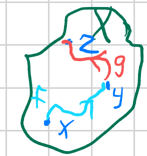
$$h(u) = \begin{cases} f(u) & \text{if } u \in A \\ g(u) & \text{if } u \in B \end{cases} \quad \text{is continuous.}$$

(If $u \in A \cap B$ then $f(u) = g(u)$ is the hypothesis)

Proof. Let $C \subseteq W$ be a closed set.
Then $h^{-1}(C) = (h^{-1}(C) \cap A) \cup (h^{-1}C \cap B)$
$= f^{-1}(C) \cup g^{-1}(C)$ is a union of two closed sets, so is closed. Therefore h is continuous. □

A similar lemma is true where A, B are open subsets. The proof verifies $h^{-1}(\text{open})$ is open.

Lemma 12.1.
(a) If f : [0,1] -> X is a path, so is $\bar{f} : [0,1] \to X$ defined by $\bar{f}(t) = f(1-t)$.
(b) If f is a path from x to y and g is a path from y to z, then then $f * g : [0,1] \to X$ is a path from x to z, defined by



$$f * g = \begin{cases} f(2t) & \text{if } 0 \le t \le \frac{1}{2} \\ g(2t-1) & \text{if } \frac{1}{2} \le t \le 1 \end{cases}$$

Proof (a) ✓ (b) $f * g$ is continuous by Lemma 12.2 with $W = [0,1]$
$A = [0, \frac{1}{2}]$ $B = [\frac{1}{2}, 1]$.

Application: R^n - several points is path connected. To show there is a path $\underline{v}$ to $\underline{w}$ use the straight line path except if some points on the path have been removed. Then choose another point $\underline{u}$ so that straight lines $\underline{v}$ to $\underline{u}$, $\underline{u}$ to $\underline{w}$ are OK. Multiply them.