Math 5345H **Worksheet 1** **9/3/2025**

Upload to Gradescope the starred questions 2, 8, 9 from Worksheet 1 before the end of the day on Wednesday 9/10/2025

1. Which of the following have you seen defined in a course before?

Notation	Yes / No ?
$X \subseteq Y$	
$X \subset Y$	
$X - Y$	
$\emptyset$	
$\{\text{something} \mid \text{condition}\}$	
$X \times Y$	
function $f : X \to Y$	
identity function	
Image $f(X)$	

Notation	Yes / No ?
$W \cup W'$	
$W \cap W'$	
Inverse image $f^{-1}(Z)$	
injective mapping	
surjective mapping	
bijective mapping	
composite gf	
composite fg	
composite $g \circ f$	

2.* Let $f : X \to Y$ be a function, and let W, W' be subsets of X. Are either of the following equalities always true?

$$f(W \cup W') = f(W) \cup f(W'), \qquad f(W \cap W') = f(W) \cap f(W').$$

If either of them is not always true, find a counterexample, and also an instance where the equation is true. If the equation is true, give a proof.

3. Let $f : X \to Y$ be a function, and let Z, Z' be subsets of Y. Are either of the following equalities always true?

$$f^{-1}(Z \cup Z') = f^{-1}(Z) \cup f^{-1}(Z'), \qquad f^{-1}(Z \cap Z') = f^{-1}(Z) \cap f^{-1}(Z').$$

Give proofs or counterexamples.

4. Let X, Y, Z be subsets of some bigger set. Are either of the following equalities always true?

$$X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z) \qquad X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$$

Give proofs or counterexamples.

5. Let A, B, C be sets and $f : A \to B$ and $g : B \to C$ be functions. Suppose that $g \circ f$ is injective. What can you say about f or g? Prove it.

6. Let A, B, C be sets and $f : A \to B$ and $g : B \to C$ be functions. Suppose that $g \circ f$ is surjective. What can you say about f or g? Prove it.

7. If $f : X \to Y$ is bijective, show that there is a function $g : Y \to X$ so that $gf = 1_X$ (the identity on X) and $fg = 1_Y$.

8.* If $f : X \to Y$ and $g : Y \to X$ are functions so that $gf = 1_X$ and $fg = 1_Y$, show that both f and g are bijective.

9.* (a) Construct a relation on the 3-element set $\{a, b, c\}$ that is not an equivalence relation.
(b) How many equivalence relations are there on $\{a, b, c\}$?

10. Let $f : A \to B$ be a surjective function. Let us define a relation on A by setting $a_0 \sim a_1$ if $f(a_0) = f(a_1)$.
(a) Show that this is an equivalence relation.
(b) Let A^* be the set of equivalence classes. Show that there is a bijective correspondence of A^* with B.