Math 5345H **Worksheet 11** **11/11/2025**

Upload to Gradescope the starred questions 1, 2b, 4a, 4d from Worksheet 11 before the end of the day on Wednesday 11/19/2025

1.* A sheet with a net to make a projective plane was distributed earlier as part of Worksheet 5.

(a) Ignoring the 6 glue flaps, label the edges of the diagram so as to indicate the identifications that are made in making this surface, showing which pairs of edges are identified, and in which direction.

(b) Use this diagram to show that the surface is a projective plane.

2. Write the projective plane as P, the Klein bottle as K, the torus as T and the 2-sphere as S. Assume the identifications $K = P\#P$ and $T\#P = K\#P$ and write T_n, P_n, K_n for the n-fold connected sums of T, P, K with themselves. Consider the surfaces that can be obtained by taking the iterated connected sum of S, P and T with each other.

(a) Show that these spaces are homeomorphic to the spaces in the list S, T_n, P_m

(b*) Show that these spaces are homeomorphic to the spaces in the list

$$S, P, K, T_n, T_n\#P, T_n\#K$$

with $n \geq 1$.

(c) Show that these spaces are homeomorphic to the spaces in the list

$$S, T_n, P, K_m, P\#K_m$$

with $m \geq 1$.

3. In class we did the 'useful move' that a polygonal region with edge labeling ay_1ay_2 is equivalent to a polygonal region with edge labeling $aay_1^{-1}y_2$, where y_1, y_2 are arbitrary words. Extend this result to show that $y_0ay_1ay_2 \sim aay_0y_1^{-1}y_2$ as follows: first break into 2 pieces: $y_0ab^{-1} \sqcup by_1ay_2$. Now justify the steps to bring this to $by_2by_1y_0^{-1}$. Finally use the useful move to get $bby_2^{-1}y_1y_0^{-1}$, then flip to get $y_0y_1^{-1}y_2b^{-1}b^{-1}$, then permute and relabel to get $aay_0y_1^{-1}y_2$.

4. Munkres' book identifies the following moves on labeled polygonal regions, of which we have seen all except the third:

(i) $y_0ay_1ay_2 \sim aay_0y_1^{-1}y_2$.

(ii) $y_0aa^{-1}y_1 \sim y_0y_1$.

(iii) $w_0y_1ay_2by_3a^{-1}y_4b^{-1}y_5 \sim w_0aba^{-1}b^{-1}y_1y_4y_3y_2y_5$.

(iv) $w_0ccaba^{-1}b^{-1}w_1 \sim w_0aabbccw_1$.

Using these transformations identify each of the following surfaces as an iterated connected sum of tori or projective planes:

(a*) $abacb^{-1}c^{-1}$

(b) $abca^{-1}cb$

(c) $abbca^{-1}ddc^{-1}$

(d*) $abcda^{-1}b^{-1}c^{-1}d^{-1}$

(e) $abcdabdc$