Math 5345H **Worksheet 5** **9/28/2025**

Upload to Gradescope the starred questions 1, 2ab, 3 from Worksheet 5 before the end of the day on Wednesday 10/8/2025

1.* Suppose that $f : X \to Y$ is a surjective mapping where X is a topological space. Suppose that Y has the quotient topology with respect to f. Show that a subset A of Y is closed if and only if $f^{-1}(A)$ is closed in X.

2.* Let $X = \{a, b, c\}$ have the topology with open sets $\emptyset, \{a\}, \{a, b\}, \{a, b, c\}$ and let $Y = \{u, v\}$ be a set. Let $f, g : X \to Y$ be the mappings $f(a) = f(b) = u$, $f(c) = v$ and $g(a) = g(c) = u$, $g(b) = v$.

(a)* How many open sets does Y have in the quotient topology determined by f? Is f an open mapping? Is f a closed mapping?

(b)* How many open sets does Y have in the quotient topology determined by g? Is g an open mapping? Is g a closed mapping?

3.* Let $f : \mathbb{R} \to S^1$ be defined by $f(t) = (\cos(2\pi t), \sin(2\pi t)) \in \mathbb{R}^2$, where S^1 denotes the circle that is the image of f. Prove that the quotient topology on S^1 determined by f is the same as the topology on S^1 induced from $\mathbb{R}^2$.

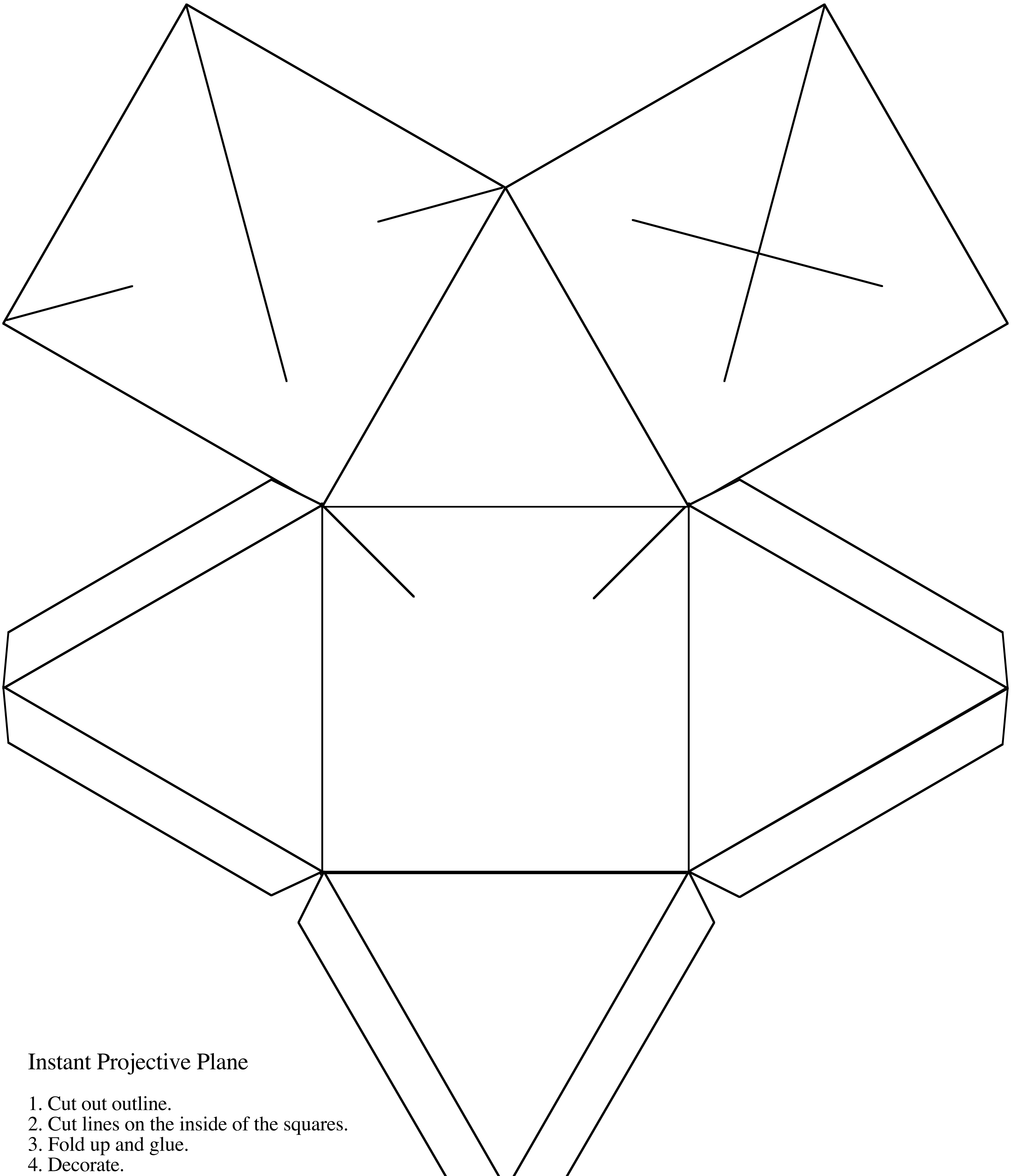
4. Prove that $\mathbb{RP}^1$ and S^1 are homeomorphic.

5. Is the following interesting?: The function $f : \mathbb{RP}^2 \to \mathbb{R}^4$ given by

$$f(\{x, -x\}) = (x_1^2 - x_2^2, x_1x_2, x_1x_3, x_2x_3)$$

is continuous and injective.

6. Consider the Möbius strip, obtained by identifying two opposite sides of a rectangle in the opposite direction (with a twist before glueing). Is the mapping f : rectangle $\to$ Möbius strip closed? Is it open?



Instant Projective Plane

1. Cut out outline.
2. Cut lines on the inside of the squares.
3. Fold up and glue.
4. Decorate.