Section 2.2: Equilibrium solutions and stability

New vocabulary:
- phase diagram
- Autonomous first order d.e. \( \frac{dx}{dt} = f(x) \), no \( t \)
- Critical points
- Equilibrium solution
- Stable, unstable

Phase diagram has critical points marked, arrows nearby to indicate the direction of the derivative.

Stable: return to the solution from nearby points

Unstable: we leave the solution from nearby points.

Stable, unstable

We don't need to know
- bifurcation point, bifurcation diagram, points, pages 91, 92, 93

We could have said something about the long term behavior of \( \frac{dx}{dt} = 4x (7-x) \) without solving the equation.

Take \( x = 6 \): \( \frac{dx}{dt} = 4 \cdot 6 \cdot 1 > 0 \)

\( x = 8 \): \( \frac{dx}{dt} = 4 \cdot 8 \cdot (-1) < 0 \)

\( x = -1 \), \( \frac{dx}{dt} = 4 \cdot (-1) \cdot (7+1) < 0 \)
Consider an equation $dx/dt = f(x)$

Solve $f(x) = 0$ to find the critical points. Then analyze the sign of $f(x)$ to determine whether each critical point is stable or unstable, and construct the phase diagram.

$$dx/dt = 3x (x-5)$$

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**Question.** For $dx/dt = x^2 (x-4)$

1. How many critical points are there? 2
2. How many are stable? 0
3. How many are unstable 1
   a. 1
   b. 2
   c. 3
   d. 0

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$x = 0$ is semistable.