Section 3.1: Linear systems

We learn:
- what is a system of linear equations?
- How to solve one by ‘elimination’
- What are elementary operations
- How to find if the system is consistent or inconsistent
- There are 0, 1, or infinitely many solutions

Page 145 question 4.
Solve \( 5x - 6y = 1 \)
\( 6x - 5y = 10 \)

Solution. We eliminate the variable \( x \)
\[
\begin{align*}
\frac{6}{5} \text{ (eqn 1)}: & \quad 6x - \frac{36}{5} y = \frac{6}{5} \\
\text{Subtract this from eqn 2: } & \quad 0x + \left( -5 + \frac{36}{5} \right) y = 10 - \frac{6}{5}
\end{align*}
\]

We use ‘back substitution’ to find \( x \). Substitute the value for \( y \) in \( 5x - 6y = 1 \)
\[
\begin{align*}
\frac{11}{5} y &= \frac{44}{5} \\
y &= 4
\end{align*}
\]

\[
x = \frac{1 + 6y}{5} = 5
\]

There is a unique solution: \( (x, y) = (5, 4) \).
We used ‘back substitution’ and some ‘elementary operations’.
Page 145 question 6.

Solve

\[ 4x - 2y = 4 \]
\[ 6x - 3y = 7 \]

The solutions are \((x, y) = \)

a. (3,2)
b. (2,3)
c. (-1,5)
d. \((1/2, -1/3)\)
e. None of the above.

Add \((-3/2)\) eqn 1 to eqn 2:

\[ 0 = 1 \]

Similarly: \[ 0 = 0, \text{ which is OK.} \]

The general solution is: \((x, 2x - 2)\)

There are infinitely many solutions.
Summary

Solve \[4x - 2y = 4\]
\[6x - 3y = 6\]

Solve \[4x - 2y = 4\]
\[6x - 3y = 7\]

Page 145 question 4.
Solve \[5x - 6y = 1\]
\[6x - 5y = 10\]
Elementary operations

1. Multiply an equation by a non-zero scalar.
2. Switch two equations.
3. Add a multiple of one equation to another.

Solve

\[
\begin{align*}
2y + 3z &= 7 \\
2x + 4y + z &= -1 \\
x + 3y + 2z &= 3
\end{align*}
\]
The equation $y'' - 121y = 0$ has general solution $y = Ae^{11x} + Be^{-11x}$.

If $y(0) = 44$ and $y'(0) = 22$, find $A$ and $B$. 