Math 8302 **Homework 13** **PJW**

Date due: Wednesday May 1, 2002.

1. Let $M = \mathbb{R}^3 \backslash \{0\}$ and define

$$F_v = \{w \in T_v(M) \bigm| w \perp v\},$$
$$E_v = \{(a, b, c) \in T_v(M) \bigm| (b, c, a) \perp v\},$$

for all $v \in M$, regarded as geometric vectors in $\mathbb{R}^3$. Prove that $F = \bigcup_{v \in M} F_v$ is an integrable 2-direction field, and describe the integral manifolds. Decide whether or not $E = \bigcup_{v \in M} E_v$ is integrable.

2. Take $M = \mathbb{R}^3$ and let F be the 2-direction field spanned by the pointwise linearly independent vector fields

$$X = \frac{\partial}{\partial x} + g(x, y)\frac{\partial}{\partial z},$$
$$Y = \frac{\partial}{\partial y} + h(x, y)\frac{\partial}{\partial z}.$$

Show that F is integrable if and only if

$$\frac{\partial g}{\partial y} = \frac{\partial h}{\partial x}.$$

Show that in this case an integral manifold of F is locally a graph $z = f(x, y)$ where $f(x, y)$ solves the system

$$\frac{\partial f}{\partial x} = g(x, y),$$
$$\frac{\partial f}{\partial y} = h(x, y).$$