Math 8302 **Homework 5** **PJW**

Date due: Wednesday February 27, 2002.

1. Rotman page 130, 6.20.

2. Rotman page 130, 6.21.

3. Let $f : \mathbb{R}^n \to \mathbb{R}^p$ be a differentiable function. Consider the mapping

$$df : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^p \times \mathbb{R}^p$$

specified by $df(u, v) = (f(u), f'(u)(v))$.
(a) Show that $d(f \circ g) = df \circ dg$.
(b) Discuss the accuracy of the equation $d(f \circ g) = df \circ dg$ which appears 12 lines from the bottom of page 8 in the book by Aubin.

4. Let $E = \mathbb{R}^n$ and $F = \mathbb{R}^p$.
(a) Let $m : \mathcal{L}(E, F) \times E \to F$ be the evaluation map $m(A, u) = A(u)$. Show that $m'_{(A,u)}(B, v) = A(v) + B(u)$.
(b) Let $f : U \to \mathcal{L}(E, F)$ be a C^k map where U is an open subset of $\mathbb{R}^q$. Show that the map $\hat{f} : U \times E \to F$ defined by $\hat{f}(x, u) = (f(x))(u)$ is C^k, and determine $\hat{f}'$.
[Hint: factorize $\hat{f}$ as $U \times E \to \mathcal{L}(E, F) \times E \to F$.]

5. Using the definition of a set of measure zero on page 9 of Aubin, write out a proof that a countable union of sets of measure zero has measure zero.