Math 8302 **Homework 6** **PJW**

Date due: Wednesday March 6, 2002. There will be a quiz on this date.

1. (a) Let E_1, E_2, E_3 be spaces $\mathbb{R}^n$, $\mathbb{R}^m$, $\mathbb{R}^p$ and let $f : E_1 \times E_2 \to E_3$ be a C^1 map. Let $a \in E_1$, $b \in E_2$ and let $f_1 : E_1 \to E_3$, $f_2 : E_2 \to E_3$ be the maps $f_1(e_1) = f(e_1, b)$, $f_2(e_2) = f(a, e_2)$. Prove that $f'(a,b)(e_1, e_2) = f_1'(a)(e_1) + f_2'(b)(e_2)$.
   (b) Now let $p = m$ as in the Implicit Function Theorem, and suppose $g : \Omega \to E_2$ is a C^1 map defined on an open subset $\Omega \subset E_1$ and that $f(x, g(x)) = c$ is a constant vector. Suppose that $a \in \Omega$, put $b = g(a)$ and suppose that f_2' has rank m at b. Show that $g'(a) = -f_2'(b)^{-1} \circ f_1'(a)$.

2. (a) Suppose that $h : \Omega \to E_2$ is a C^1 map defined on an open subset $\Omega \subset E_1$ and that $h' = 0$. Show that h is the constant function.
   (b) In the situation of question 1(b) suppose that g and $\tilde{g}$ are both C^1 functions mapping Ω to E^2 so that $f(x, g(x)) = f(x, \tilde{g}(x)) = c$ is a constant and with $g(a) = \tilde{g}(a) = b$. Suppose that the function $\tilde{f}$ defined by $\tilde{f}(y) = f(x, y)$ has rank m at y for all choices of x and y. Prove that $g = \tilde{g}$.

3. 0.45 on page 16 of Aubin.

Make sure you can do questions such as the following:

A. Let $\det : M_n(\mathbb{R}) \to \mathbb{R}$ be the determinant map. Show that $\det'(I) = \text{trace}$, where I is the identity matrix.