Math 8302 **Homework 9** **PJW**
Date due: Wednesday April 3, 2002.

1. Let $f : M \to N$ be an immersion and let $W \subseteq M$ be a submanifold. Show that $f|_W : W \to N$ is an immersion.

2. (a) Define $f : S^2 \to \mathbb{R}^4$ by the formula

$$f(x, y, z) = (yz, xz, xy, x^2 + 2y^2 + 3z^2).$$

Prove that f passes to a well-defined embedding $\bar{f} : \mathbb{R}P^2 \to \mathbb{R}^4$.

3. (Manifolds and Topology exam, Fall 2001) Let S be the set of all polynomials in one real variable x with rational coefficients. Let T be the union of all the graphs in the plane of the polynomials belonging to S. Prove that $\mathbb{R}^2 - T$ is non-empty.

4. (Stack of Records Theorem.) Suppose that y is a regular value of $f : M \to N$ where M is compact and has the same dimension as N. Show that $f^{-1}(y)$ is a finite set $\{x_1, \ldots, x_n\}$. Prove that there exists a neighborhood U of y in Y such that $f^{-1}(U)$ is a disjoint union $V_1 \cup \cdots \cup V_n$ where V_i is an open neighborhood of x_i and f maps each V_i diffeomorphically onto U. [In other words, the set of regular values is evenly covered by f. Hint: Pick disjoint neighborhoods W_i of x_i that are mapped diffeomorphically. Show that $f(M - \bigcup W_i)$ is compact and does not contain y.]