Math 8201 **Homework 1** **PJW**

Date due: September 16, 2002

Hand in only the five starred questions.

Section 1.1 nos. 14, 25*, 32, 34, 36.

Section 1.2 nos. 4*, 5, 7, 11.

A. Let α be a rotation about the origin in the plane and let ρ be the reflection in the x-axis. Show that $\rho\alpha\rho^{-1} = \alpha^{-1}$.

B. * Consider a pentagonal prism as shown, and let G be the group whose elements are the rotations of 3-dimensional space which leave the prism looking the same after doing the rotation as it looked before.
(a) What is the order of G?
(b) How many elements of order 2 does G have?
(c) Show that G is not commutative.

C. Let n be a positive integer, and let G be the group whose elements are the $2n$ symbols $x^i y^j$ where $i \in \{0, 1\}$ and $j \in \{0, \ldots, n-1\}$, subject to the rules $x^2 = y^n = e$ and $xy = y^{-1}x$. Express yx in the form $x^i y^j$ with $i \in \{0, 1\}$ and $j \in \{0, \ldots, n-1\}$.

Section 1.3 nos. 15, 19.

Section 1.4 no. 11.

D. * Show that the group $H(\mathbb{Z}/2\mathbb{Z})$ of 1.4 no. 11 is isomorphic to D_8.

Section 1.5 no. 3* (proving that you have indeed found generators and relations).