Math 8201 **Homework 6** **PJW**

Date due: October 21, 2002. There will be a quiz on this day.

Section 4.1 10

V. Let G be the group of *all* isometries of a regular tetrahedron:

Show that $G \cong S_4$. (Hence G is isomorphic also to the group of rotations of the cube.)

Section 4.2 2, 7*, 8*, 9*, 14

Section 4.3 2, 4, 5, 6*, 9, 10, 11, 13, 25, 29, 30, 31, 32, 34 (There are many worthwhile questions to look at in this section!)

W. Let G be an infinite group containing an element $x \neq 1$ having only finitely many conjugates. Prove that G is not simple.

X. Let $a = (1,2,3,4) \in S_4$ and put $N(a) = \{g \in G \mid ag = ga\}$. Determine what $N(a)$ is, as a subgroup of S_4. (For example, describe its structure and its order.)

Y. Show that when $a = (4,5) \in S_5$, the subgroup $N(a)$ consists of $S_3 \cup S_3 a$, where S_3 denotes the symmetric group on three symbols as a subgroup of S_5 permuting the symbols $\{1,2,3\}$.

Z. (like qn. 10) Consider the permutation $a = (1,2,3,4,5)$.
(a) Show that a has 24 conjugates in S_5.
(b) Show that a has only 12 conjugates in A_5. (Hint: compute the index of $N(a)$ in A_5.)
(c) Show that $(1,2,3,4,5)$ is conjugate in A_5 to $(5,4,3,2,1)$.
(d) Show that $(1,2,3,4,5)$ is not conjugate in A_5 to $(1,3,5,2,4)$.

AA. Let $a = (1,2,3,4)(5,6,7) \in S_7$.
(a) Find a permutation g of the symbols $\{1,2,3,4,5,6,7,8\}$ so that

$$g(1,2,3,4)(5,6,7)g^{-1} = (2,1,6,5)(3,8,7).$$

Express g as a product of disjoint cycles.
(b) Calculate the number of conjugates of a in S_7. Calculate the number of conjugates of a in S_8.
(c) Show that the only elements of S_7 which commute with a are the powers of a.