Math 8201 **Homework 8** **PJW**

Date due: November 4, 2002. There will be a quiz on this date.

Hand in only the starred questions.

Section 4.5 8, 10, 11*, 14, 15, 17*, 18, 21, 30, 32, 33*, 34, 35, 41, 42*

There are many questions on Sylow's theorem in the preliminary written exams from past years. For example, the question which appeared as question 5 in the Spring of 1994 is challenging.

BB*. Let G be a finite group and let $O_p(G)$ denote the unique largest normal p-subgroup of G, which contains all other normal p-subgroups of G and whose existence was established in a previous homework question. Show that $O_p(G)$ equals the intersection of all the Sylow p-subgroups of G:

$$O_p(G) = \bigcap_{P \text{ a Sylow } p-\text{subgroup}} P.$$

CC. Let G be a finite group and H a subgroup. Let P_H be a Sylow p-subgroup of H. Prove that there exists a Sylow p-subgroup P of G such that $P_H = P \cap H$.

DD. Prove that one of the Sylow subroups of a group of order 40 is normal. Give an explicit example of a group of order 40 in which the Sylow 2-subgroup is not normal.

Section 4.6 2*, 3, 4, 5, 6

EE. Let G be a non-abelian simple group, and H a subgroup of G with $H \neq G$. Let $A = \{xH \mid x \in G\}$ be the set of left cosets of H in G and let $\pi : G \to S_A$ be the action of G on A given by left multiplication, so that $\pi(g)(xH) = gxH$.

(a) Show that $\operatorname{Ker}\pi = \{e\}$.

(b) Show that if H has finite index in G then G itself must be finite.

(c) Suppose that G is indeed finite and that $|G : H| = n$. Show that G is isomorphic to a subgroup of A_n. (Hint: consider $\pi(G) \cap A_n$.)