Math 8202 **Homework 10** **PJW**

Date due: April 14, 2003. There will be a quiz on this date.

Hand in only the starred questions.

Section 14.1, page 546 5, 6*, 7, 8*, 9*, 10.

Section 14.2, page 562 3, 4*, 5*, 6.

J. (Fall 2001, qn. 5) (15%) Let $f(x) = x^4 + ax^3 + bx^2 + ax + 1 \in \mathbb{Q}[x]$, and let E be the splitting field of f.
   (a) (3) Show that for each root α of f in E, also α^{-1} is a root of f.
   (b) (5) Show that $[E : \mathbb{Q}] \leq 8$.
   (c) (7) Show that if f is irreducible in $\mathbb{Q}[x]$ and has exactly two real roots and two complex roots, then the Galois group $G_{E/\mathbb{Q}}$ is isomorphic to D_4, the dihedral group of order 8.

K. (Spring 1995, qn. 3) (15%) Let $f(X) \in k[X]$ be a polynomial of degree n over k and let K be a splitting field for f over k. Suppose that the Galois group $\mathrm{Gal}(K/k)$ is the symmetric group S_n.
   (a) (7) Show that f is irreducible in $k[X]$, and separable.
   (b) (5) Let α be a root of f in K. Show that in $k(\alpha)[X]$, f factorizes as

$$f(X) = (X - \alpha)g(X)$$

   where $g(X)$ is an *irreducible* polynomial.
   (c) (3) Determine the Galois group of g over $k(\alpha)$.