Math 8202 **Homework 11** **PJW**
Date due: April 21, 2003.

Hand in only the starred questions.

Section 14.2, page 561 Hand in 11*, 14*, 28*.
All of the questions 6 – 16 are worth looking at.

Section 14.3, page 569 Hand in 10*.
We have pretty much covered the material in Section 14.3 already. If you glance at questions 1, 2, 3, 4, 5, 8, 9 from Section 14.3 you should find you know how to do them, and sometimes you will recognize things from other sections that we have done recently, e.g. in question 8.
The graduate written exam next week: Apart from finishing off Galois theory, there remain two significant things which we have not yet covered which are on the syllabus, and they are the material on projective modules, and the spectral theorem for Hermitian and symmetric matrices. You can read about projective modules etc. in section 10.5 of Dummit and Foote. The spectral theorem does not appear in Dummit and Foote, but the account in Chpater 15 of Lang's book, up to section 7 of that chapter, is not bad.

L. (Fall 2002, qn. 6) Let a be a nonzero rational number.
  (a) (6%) Determine the values of a such that the extension $\mathbb{Q}(\sqrt{ai})$ is of degree 4 over $\mathbb{Q}$, where $i^2 = -1$.
  (b) (12%) When $K = \mathbb{Q}(\sqrt{ai})$ is of degree 4 over $\mathbb{Q}$ show that K is Galois over $\mathbb{Q}$ with the Klein 4-group as Galois group. In this case determine all the quadratic extensions of $\mathbb{Q}$ contained in K.

M*. (Spring 2002, qn 5) Let F be a field of characteristic 0. Let $f(x) = x^n - a \in F[x]$, and assume that f does not have a root in F. (But it is *not* assumed that $f(x)$ is irreducible in $F[x]$.) Finally, let E be a splitting field of $f(x)$.
  (a) (6%) Show that if F contains a primitive n^{th} root of unity, then the Galois group $G_{E/F}$ is isomorphic to a subgroup of the additive group $\mathbb{Z}/n\mathbb{Z}$.
  (b) (6%) Show that if F contains a primitive n^{th} root of unity, then all of the irreducible factors of $f(x)$ in $F[x]$ are of the same degree.
  (c) (6%) Show that if F contains a primitive n^{th} root of unity and $g(x)$ is an irreducible factor of $f(x)$ in $F[x]$, then $g(x) = x^k - b$, where k is a divisor of n, b is an element of F, and $b^{n/k} = a$.
  (d) (6%) Now let $f(x) = x^6 + a$ in $\mathbb{R}[x]$, where $a > 0$. Determine whether each of the following two statements is true or false:
  (i) All of the irreducible factors of $f(x)$ in $\mathbb{R}[x]$ have the same degree.
  (ii) If $g(x)$ is an irreducible factor of $f(x)$ in $\mathbb{R}[x]$ then $g(x) = x^k - b$, where k is a divisor of 6 and $b \in \mathbb{R}$ satisfies $b^{6/k} = a$.