Math 8202 **Homework 6** **PJW**

Date due: March 9, 2003. There will be a quiz on this date.

Hand in only the starred questions.

Section 12.3, page 479 nos. 7b*, 9*, 16*, 18*, 24*. All of questions 4 – 39 are good (omitting the parts of questions such as in question 11 which have to do with the rational canonical form).

Section 15.1, page 646 nos. 1, 2, 3, 4, 5*, 9*, 11, 12

The algorithm presented in the book on pages 476 – 479 for putting a matrix into Jordan canonical form (and finding a basis which achieves this) first puts the matrix into rational canonical form. This probably works quite well and fits with our work on Smith normal form, but it is not the algorithm I would recommend, which is technically more elementary. The algorithm I will teach to find an appropriate basis for the generalized eigenspace for an eigenvalue λ of a matrix A first finds the smallest n such that $(A-\lambda I)^n = 0$. Now find a basis for the whole space modulo the nullspace of $(A-\lambda I)^{n-1}$. Apply $(A-\lambda I)$ to these elements and extend them to a basis of the nullspace of $(A-\lambda I)^{n-1}$ modulo the nullspace of nullspace of $(A-\lambda I)^{n-2}$. Apply $(A-\lambda I)$ to these elements and extend them to a basis of the nullspace of $(A-\lambda I)^{n-2}$ modulo the nullspace of nullspace of $(A-\lambda I)^{n-3}$. And so on. Eventually we will have a basis of the generalized eigenspace with respect to which A is in Jordan canonical form.

Remember in all this that it is already laborious to do this with a 3×3-matrix, and about the most that could be done in exam conditions is a straightforward 4×4-matrix, or a matrix which has a special structure.