Math 8202 **Homework 9** **PJW**

Date due: April 7, 2003.

Hand in only the starred questions.

Section 13.4, page 525 3*, 4.

Section 13.5, page 531 5*, 6*, 7, 8, 10*.

F*. (Fall 2002 qn. 5, part (a)) Let k be a field of characteristic $p > 0$, and $K = k(t)$ where t is an element transcendental over k. Show that $X^p - t$ is irreducible in $K[X]$.

G*. (Fall 2001, qn. 6) (10%) Let $\mathbb{F}_{p^k}$ be the field with p^k elements, where p is prime.

(a) Show that $x^4 + 1 \in \mathbb{F}_p[x]$ has a root in $\mathbb{F}_{p^2}$.

(b) Deduce that $x^4 + 1$ is reducible in $\mathbb{F}_p[x]$. For which values of p does a linear factor exist in $\mathbb{F}_p[x]$?

[You may assume standard facts about finite fields.]

H. (Fall 2000, qn. 5)(12%) Let $K \supseteq k$ be a field extension and $f \in k[X]$ an irreducible polynomial of degree relatively prime to the degree of the field extension $[K : k]$. Show that f is irreducible in $K[X]$.

I. (Fall 2000, qn. 6)(15%) a) (8) Let $K \supseteq k$ be a field extension of prime degree, and let $a \in K$ be an element which does not lie in k. Considering K as a vector space over k, let $m_a : K \to K$ be the k-linear mapping specified by $m_a(x) = ax$. Prove that the characteristic polynomial of m_a is irreducible.

b) (7) Let α be a root of $X^3 - X + 1$ in $\mathbb{F}_{27}$. Find the minimal polynomial of α^4 over $\mathbb{F}_3$.

[Here $\mathbb{F}_{27}$ and $\mathbb{F}_3$ denote fields with 27 and 3 elements, respectively. You may assume that $X^3 - X + 1$ is irreducible in $\mathbb{F}_3[X]$.]