Math 1031 Practice Exam 2
November 2004

There are nine questions, worth varying percentages as shown. Show your work in the space provided. You may not use your books or notes or a graphing calculator on this exam. You may use a regular scientific calculator.

1. A ladder ten feet long is placed agains a wall so that the height of the top of the ladder above the ground is two feet more than the distance of the foot of the ladder from the wall. How high is the top of the ladder above the ground?

Solution: Let x be the height of the top of the ladder above the ground. The distance of the foot from the wall is $\sqrt{10^2 - x^2}$ so $x = \sqrt{10^2 - x^2}+2$. We solve this. $x-2 = \sqrt{10^2 - x^2}$, $(x-2)^2 = 100 - x^2$, and solving the quadratic gives $x = 8$ or -6. Since -6 makes no sense for this problem the top of the ladder is 8 feet above the ground.

2. Solve the following inequalities, specifying the region of real numbers which represents the solution:
   (a) $\dfrac{1}{x} > \dfrac{1}{x+1}$
   (b) $|2x - 1| \leq 5$.

Solution (a) $\frac{1}{x} - \frac{1}{x+1} = \frac{1}{x(x+1)} > 0$. The critical points are $x = 0, -1$. We test $x = -10, -\frac{1}{2}, 1$ to get that the solution is $(-\infty, -1) \cup (0, \infty)$.
Solution (b) $-5 \leq 2x - 1 \leq 5$, so $-4 \leq 2x \leq 6$ and $-2 \leq x \leq 3$.

3. Find all real solutions to the equation $x^4 - x^2 - 2 = 0$.

Solution: $x^2 = \frac{1 \pm \sqrt{1-4(-2)}}{2} = \frac{1 \pm 3}{2} = 2$ or -1. Thus $x = \pm\sqrt{2}$, since $\sqrt{-1}$ makes no sense here.

4. Solve the equation:
$$3 - \sqrt{2x - 4} = 0$$

Solution $3 = \sqrt{2x - 4}$, $9 = 2x - 4$, $2x = 13$, $x = 6.5$.

5. Find the center and radius of the circle $2x^2 + 2y^2 + 12x + 16y = 0$.
   Answer: the center is at $\underline{(-3, -4)}$, and the radius is $\underline{5}$.

6. Determine whether or not the points $(1, 2)$, $(5, 3)$, $(6, -1)$, $(2, -2)$ are the vertices of a square.

Solution One way to do this is to verify that each of the four distances between the pairs of points taking them in the order given, as though they were the vertices of a square, is $\sqrt{17}$. From this we may deduce that the points are the vertices of a rhombus. We may verify that one of the angles is a right angle (this is enough) by finding the slope of the line joining $(1, 2)$ and $(5, 3)$ (it is $\frac{1}{4}$) and the slope of the line joining $(5, 3)$ and $(6, -1)$ (it is -4). Since $-4(\frac{1}{4}) = -1$ we deduce that these lines are perpendicular. Therefore the figure is a square.

7. Find the equation of the line which passes through the point $(3, 5)$ and is perpendicular to the line through $(1, -1)$ and $(4, -2)$.

Solution. The slope of the line through $(1, -1)$ and $(4, -2)$ is $\frac{-1-(-2)}{1-4} = \frac{1}{-3} = -\frac{1}{3}$. The desired line is $y - 5 = 3(x - 3)$.

8. A computer bought today for \$1,200 goes down in value according to a linear model so that after 4 years it is worth \$300.
a) Write down an equation expressing the value V of the computer in terms of the time t, where t is expressed in years and $t = 0$ corresponds to the moment the computer is bought.
b) According to this model, how much will the computer be worth 5 years after it is bought?

Solution (a) The rate of change of price is $\frac{300-1200}{4} = -225$. The equation is $V - 1200 = -225t$, or $V = -225t + 1200$.
(b) When $t = 5$ we have $V = 1200 - 1125 = \$75$.

9. For each of the graphs (a) - (f) on page 107 of the book LHH say whether it has
A: symmetry with respect to the x-axis,
B: symmetry with respect to the y-axis,
C: symmetry with respect to the origin, or
D: none of these symmetries.
Finally, match the graphs to the functions in questions 29 - 34.
[On the actual exam, I would not refer to questions in the book. I do it here because it is hard for me to include graphics in this document, and it saves me work to refer to the book.]

Solution. a D 32, b B 34, c D 29, d B 31, e C 33, f D 30.