Answers to Practice Exam 3

1. a even, b odd, c even, d odd, e neither.

2.
$$(f \circ g)(x) = \sqrt{\frac{1}{1+x^2} - 1} = \sqrt{\frac{-x^2}{1+x^2}}.$$

   However this doesn't make much sense, because this function is undefined everywhere.

$$(g \circ f)(x) = \frac{1}{x}.$$

3. Use the relationship $4x + 3y = 200$ to substitute for x in the expression $2xy$ for the area. This gives a quadratic expression for the area which we maximize. The maximum occurs when $x = 25$ and $y = 100/3$.

4. The equation $y = a(x-1)(x-3)$ has the correct x-intercepts for every choice of a. The vertex has x-coordinates half-way between the x-intercepts (we could also get this by completing the square for the quadratic function), which is at $x = 3$. Substituting $x = 3$ and $y = 5$ gives $a = -5$ so $y = -5(x-1)(x-3)$ is the equation.

5. The vertex is at $(-1, -9)$ and the intercepts are $(0, -7)$, $(1/2, 0)$, $(-5/2, 0)$.

6. It is $(-1, 1) \cup (1, \infty)$.

7. (a) the range of g is $[-1, 1/2]$.
   (b) The graph of $f + g$ passes through $(0, 1/2), (1, 3/2), (2, 3/4), (3, 1)$ with straight lines inbetween.
   (c) The graph of $f \circ g$ passes through $(0, 1/2), (1, 0), (2, 0), (3, 1/2)$ and if you plot these points you would get full credit. It also passes through $(3/2, 1/2)$ and there are with straight lines joining these 5 points making a W shape.
   (d) The domain of $f \circ g$ is $[1, 3]$.
   (e) Shift the graph of g up 3 units and to the right 1 unit.

8. 1e 2c 3g 4d 5f 6h 7a 8b.

9. (a) No inverse exists by the horizontal line test: the x-axis meets the curve at 3 points.
   (b)
$$f^{-1}(x) = \sqrt[3]{\frac{1-x}{x}}.$$

10. (a) $x = -4, 0, 2$.
    (b) When x is large and positive so is $f(x)$. When x is large and negative so is $f(x)$, because the leading coefficient is > 0 and the degree of f is odd.
    (c) There are two turning points. Since f has degree 3 there are at most 2. Since f has 3 zeros there are at least 2.