Mathematics 1031 Formulas

Interest

Simple Interest: $A = P(1 + rt)$
Compound Interest: $A = P(1 + \frac{r}{n})^{nt}$
where P is the principal, r is the annual interest rate expressed as a decimal, n is the number of times per year the interest is compounded, A is the balance after t years.
Continuous Compounding: $A = Pe^{rt}$

Enumeration

Fundamental Counting Principle: the number of ways to perform independent tasks $T_1, \ldots, T_k$ where there are n_i ways to perform T_i is the product $n_1 \cdots n_k$.

$$n! = n(n-1)(n-2)\cdots 3 \cdot 2 \cdot 1 = n \cdot (n-1)!$$

$$P(n,k) = n(n-1)\cdots(n-k+1) = \frac{n!}{(n-k)!}$$

$$C(n,k) = \frac{n(n-1)\cdots(n-k+1)}{k(k-1)\cdots 3 \cdot 2 \cdot 1} = \frac{n!}{(n-k)!\,k!} = C(n, n-k)$$

Probability

A sample space S consists of outcomes $s_1, \ldots, s_n$. Each outcome s_i is assigned a probability p_i with

$$0 \leq p_i \leq 1 \quad \text{and} \quad p_1 + \cdots + p_n = 1.$$

The probability of an event E is the sum of the probabilities of the outcomes in E. When all the outcomes are equally likely $p_i = \frac{1}{n}$ and $P(E) = \frac{|E|}{|S|}$.
It is always true that

$$P(E \cup F) = P(E) + P(F) - P(E \cap F).$$

If E and F are *mutually exclusive* then $P(E \cup F) = P(E) + P(F)$.
If E and F are *independent* then $P(E \cap F) = P(E)P(F)$.
Also $P(E) + P(E^c) = 1$ where $E^c = S - E$ is the *complement* of E.
In independent experiments where $P(\text{success}) = p$ and $P(\text{failure}) = 1 - p$ we have

$$P(k \text{ successes in } n \text{ experiments}) = C(n,k)p^k(1-p)^{n-k}.$$

If E and F are events from the same experiment: $P(E) = P(E|F)P(F) + P(E|F^c)P(F^c)$.
The expected value to you of a game in which you win w_i when s_i occurs is

$$E = w_1 \cdot P(s_1) + w_2 \cdot P(s_2) + \cdots + w_n \cdot P(s_n).$$

Logarithms

$\log_a(uv) = \log_a(u) + \log_a(v)$, $\log_a(u^n) = n\log_a(u)$, Base Change: $\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$.