Math 5594 **Homework 1, due Monday September 18, 2006** **PJW**

1. We know that if H and K are subgroups of G then $H \cap K$ is a subgroup of G. Show by a specific example that $H \cup K$ need not be a subgroup of G. [We know that if H and K are subgroups of G then $H \cap K$ is a subgroup of G.]
2. Show from the axioms that the identity element of a group is unique; that is, if e and e^* are both elements of a group G for which $ex = xe = x$ and $e^*x = xe^* = x$ for every $x \in G$ then $e = e^*$.
3. Show from the axioms that each element x of a group G has only one inverse; that is, if $xx^{-1} = x^{-1}x = e$ and $xx^* = x^*x = e$ then $x^{-1} = x^*$.
4. Let $g \in G$. Show from the axioms that $(g^2)^{-1} = (g^{-1})^2$.
5. Write out the multiplication table for the group generated by the matrices $A = \begin{pmatrix} -1 & -1 \\ 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and show that it is 'the same' as the multiplication table of the group S_3 (which is generated by $(1, 2, 3)$ and $(1, 2)$).
6. Find all of the subgroups of the symmetric group S_3. Make a table that shows how these subgroups are contained in one another.
7. Find all of the subgroups of the cyclic group of order 6. Make a table that shows how these subgroups are contained in one another.

Some extra questions - do not hand in!

8. Let $p = (1, 2, 3)(4, 5)$ and $q = (1, 3, 5)$. Calculate the powers of p and of q, and find the products pq, qp, p^2q, p^3q and q^2p. Guess how big the group generated by p and q is (but do not try to prove it unless you can see immediately how to do it).
9. Show that if we allow elements to be called by different names then there are really only two groups of order 4, one of which is cyclic of order 4 and the other of which has no elements of order 4. Write out the multiplication tables for these groups.
10. Write out the multiplication table for the group generated by the matrices $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$.
11. Find all of the subgroups of the non-cyclic group of order 4. Make a table that shows how these subgroups are contained in one another.