

Assignment 13 - Not to be handed in. The third midterm exam is on December 4. In this exam you will be tested on the material in Sections 2.1 - 2.6 which we have studied.

The final exam will take place on Friday December 12, 1:30-4:30, and I am still not sure what room it will be in. This is to be announced. The final exam will be on everything we have done this semester, including Section 2.7 and the parts of Section 2.8 which we will study. Although no further homework is due, I give questions relevant for these sections so that you can practice.

Read: From Hubbard and Hubbard Section 2.8 only pages 232-237, the beginning of Example 2.8.6, the beginning of Example 2.8.14 and the beginning of Example 2.8.15. We will not do Kantorovich's theorem in Section 2.8.

Exercises: 2.7 (page 232): 2, 3, 5, 6
 2.8 (pages 250-252): 2, 4, 6b, 7a, 10, 12a
 2.11 (page 281): 22b, 23

There are other questions from 2.11 which are relevant, and some of these are listed on Assignment 12. There will be a further review sheet for the final exam.

Extra questions:

1. For each of the following matrices, representing a linear map $\mathbb{R}^2 \rightarrow \mathbb{R}^2$, either find a basis consisting of eigenvectors or else show that such a basis cannot exist.

$$\begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix} \quad \begin{pmatrix} 22 & -9 \\ 49 & -20 \end{pmatrix} \quad \begin{pmatrix} 4 & -1 \\ 2 & 1 \end{pmatrix} \quad \begin{pmatrix} 13 & -6 \\ 0 & -13 \end{pmatrix}$$

2. Regarding each of the matrices in question 1 as the matrix of a linear map expressed with respect to the standard basis of \mathbb{R}^2 , find the matrix of this linear map when expressed with respect to the basis $\begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \end{pmatrix}$.

3. Find a matrix P so that PAP^{-1} is diagonal, where $A = \begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix}$.

4. Find a basis for \mathbb{R}^3 consisting of eigenvectors of the following matrix, or else show that such a basis does not exist:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & 2 & 0 \end{pmatrix}$$

Comments: The culmination of Chapter 2 is the discussion of the implicit and inverse function theorems in Section 2.10. We will study them next semester. These theorems have theoretical importance in that they allow us to deduce the equivalence of two different definitions of a manifold, and they appear here in the book because at the start of Chapter 3 we define manifolds. Usually in a course at this level these theorems are stated but not proved. Our book does provide a proof, but sections of it are relegated to the Appendix, and in any case I think a more streamlined proof could be given.

We will not do Kantorovich's theorem in Section 2.8. That theorem does use the important notion of a Lipschitz condition, and it is a pity not to know about that, but I think we should move on. Section 2.9 on superconvergence we will not do at all.