

3593 Exam 2 practice questions

Sections of the book to be tested: 3.5, 3.6, 3.7, 4.1, 4.3, 4.5 and 4.8

1. Prove that a subset of a set of volume zero has volume zero.
2. Consider the functions defined on \mathbb{R}^2

$$f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{cases} 1 & \text{if } y = x^2 \text{ and } -1 \leq x \leq 1 \text{ is rational,} \\ 0 & \text{otherwise} \end{cases},$$

$$g\begin{pmatrix} x \\ y \end{pmatrix} = \begin{cases} xy & \text{if both } x \text{ and } y \text{ are rational between } -1 \text{ and } 1, \\ 0 & \text{otherwise.} \end{cases}$$

Do $\int_{\mathbb{R}^2} f |d^2x|$ and $\int_{\mathbb{R}^2} g |d^2x|$ exist? If so, what are their values?

3. Let σ be the permutation $\sigma(1) = 2, \sigma(2) = 3, \sigma(3) = 4, \sigma(4) = 1$ and let τ be the permutation $\tau(1) = 1, \tau(2) = 4, \tau(3) = 3, \tau(4) = 2$. What is the sign of the permutation $\sigma\tau$?

Relevant questions from the book:

Section 3.5 page 341: questions from assignment 4 and

Section 3.6 page 349: questions from assignment 4 and the questions listed below from Section 3.9.

Section 3.7 page 366: questions from assignment 5 and nos 21, 22 from Section 3.9 below.

Section 3.9 page 386: 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 25

Section 4.1 page 405: 10, 14, 15

Section 4.3 page 427: 5

Section 4.5 page 445: 7, 8, 11, 12, 14, 15, 16, 18

Section 4.8 page 474: 2, 12, 13, 15

Section 4.12 page 514: 11, 12, 13