Math 8201 **Homework 7** **PJW**

As well as Section 5.3, pages 212 - 215 from Section 4.1 about solvable groups are relevant this week.

Date due: October 27, 2008 Hand in the 5 starred questions.

Section 5.3 5.33, 5.34*, 5.39*, 5.40, 5.43*, 5.44, 5.45*, 5.46,

MM Calculate the commutator subgroup G' (in particular determining its order) when $G = D_{10}$ and $G = D_{12}$. List *all* composition series for these groups.

NN The definition of a *characteristic* subgroup is given in Exercise 5.19 on page 277, which also describes a property of the commutator subgroup. Show that if $H \le K \lhd G$ are subgroups and if H is a characteristic subgroup of K then $H \lhd G$.

OO* Let G be a solvable group.

(i) Prove that if H is a nontrivial normal subgroup of G then there is a nontrivial subgroup A of H with $A \lhd G$ and A abelian.

(ii) Show that G has a chain of subgroups $1 = N_0 \le N_1 \le \cdots \le N_t = G$ for which $N_i \lhd G$ and N_i/N_{i-1} is abelian for all i.

(iii) Show that if G is finite then every minimal normal subgroup of G is abelian.

(iv) Deduce that if G is finite and K is a minimal normal subgroup of G then K is a p-group for some prime p and that $x^p = e$ for every $x \in k$. (Such a group K is a vector space over the field with p elements and so is in fact a direct product of cyclic groups of order p.)