

## Assignment 9 - Due Thursday 11/08/2012

**Read:** Hubbard and Hubbard Section 2.1, 2.2 and 2.3.

**Exercises:**

Hand in only the exercises which have stars by them.

Section 2.1: 2a\*.

Section 2.2: 2e\*, 3b, 6\*(for part b please identify completely the values of a for which there is a unique solution, no solution, and infinitely many solutions), 9\*, 10. In questions which ask you to solve a system of equations which has infinitely many solutions, find an expression for the general form of the solution.

Section 2.3: 2, 3, 3b\*, 4, 5, 6\*, 7, 8, 9, 11

Extra questions:

1. Express the vectors (1,0) and (0,1) as linear combinations of (1,2) and (2,3) by solving an appropriate system of equations for the coefficients of the combinations.

2. Express the vector (5,0,1,2) as a linear combinations of (1,2,1,0) and (2,-1,0,1).

3. In each of the following, determine whether or not the vector  $v$  is a 'linear combination' of the other vectors given, that is, whether  $v$  can be expressed as a sum of terms which are a scalar times  $a$ ,  $b$  or  $c$ :

(a)  $v = 2i + 3j$ ;  $a = 2i - j$ ,  $b = 2i + j$ .

(b)\*  $v = 2i + 3j + 4k$ ;  $a = 2i - j$ ,  $b = i + j + k$ ,  $c = j - 2k$ .

(c)  $v = (3, -1, 0, -1)$ ;  $a = (2, -1, 3, 2)$ ,  $b = (-1, 1, 1, -3)$ ,  $c = (1, 1, 9, -5)$ .

4. Solve the systems

$$(a) \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} x = \begin{pmatrix} 6 \\ 5 \\ 4 \\ 3 \\ 2 \end{pmatrix}. \quad (b) \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 1 & 2 & 3 & 4 \\ 1 & 0 & -1 & -2 & -3 \\ 0 & 0 & 0 & 1 & 2 \\ 2 & 3 & 4 & 6 & 8 \end{pmatrix} x = \begin{pmatrix} 6 \\ 5 \\ -4 \\ 3 \\ 10 \end{pmatrix}.$$

5\*. Express the matrices in question 2.3.2b and 2.3.2c as products of elementary matrices.

6. Suppose that  $f : R^n \rightarrow R^n$  is a linear mapping between spaces of the same dimension. Prove that if  $f$  is 1-1 then  $f$  is onto. Prove conversely that if  $f$  is onto then  $f$  is 1-1.

**Comments:**

We have an exam on November 8, and hence no quiz. The material which will appear on the exam will be taken from Sections 1.5 - 1.10, starting in Section 1.5 at Definition 1.5.12 where limits, continuity etc. of sequences of vectors and vector-valued functions are introduced. The exam may also test you on material which has appeared in the extra questions on the assignment sheets. You may not use books or notes on the exam, but you may use a calculator.