Math 3593H Honors Mathematics II Spring Semester 2018

Assignment 2 - Due Thursday 2/1/2018

Read: Hubbard and Hubbard Section 3.1. The examples about linkages (3.1.8, 3.1.9 and 3.1.15) get into some detail that is not relevant for the course, and which you can omit. I will not test you on things to do with linkages. Skip over these examples. Also, I will not test you on Theorem 3.1.16 or Corollary 3.1.17. What you need to know from Section 3.1 are Definition 3.1.2 (graphs of functions), the equivalent Theorem 3.1.10 (zero locus), Definition 3.1.18 (parametrizations) and the examples to do with these three things.

We might get on to Section 3.2.

Exercises:
Section 3.1: 1, 2, 3, 4, 5*, 7*, 8, 11*, 13, 14, 15, 16, 19*, 22, 23, 24, 25
Section 3.2: 1*, 2, 3, 4, 5a, 6*, 8, 9
Extra questions:
A. Find the equation for the tangent line at the point (2,4,8) to the curve parametrized in question 3.1.11a
B*. Find the equation for the tangent plane at the point (sin(2)+1, 3, 2) to the surface parametrized in question 3.1.25.

Comments:
The idea of a smooth manifold is intuitively not difficult, but making it precise in mathematical terms appears to be quite complicated, according to Section 3.1. Not only that, but although you might think Section 3.1 is giving you the complete picture, in fact it is not: there is still more to the story which will not be told in this course.

The definition in 3.1 specifies that a manifold should be given as a subset of some ambient space. In general manifolds are defined more abstractly without specifying a particular embedding of the manifold into a larger space. Although the abstract definition is actually easier to work with in some ways, it does raise the question of whether every manifold can be embedded in R^n, and the definition of the tangent space becomes more difficult. The book avoids these issues, and rightly so. The course where this approach is taught is the graduate level course, 'Manifolds and Topology'.

As it is, we need to be able to work with manifolds given in three different ways: as a set which is locally the graph of a differentiable function, by means of parametrizations, and also as the set of solutions of an equation.