Math 3592 review for exam 1

On the exam points in $\mathbb{R}^3$ are sometimes written as (for example) $(1, 2, 3)$, rather than $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$. This sheet is supposed to help you by providing some extra practice. It should not be interpreted that what is on this sheet is the only practice you should do, or that questions like the ones here are the only kind you will be asked. You are advised to review the material we have covered more broadly than what is on this sheet.

1. Let L_1 be the line passing through the point $(1, 2, 3)$ perpendicular to the plane P_1 with equation $3x - y + 2z = 4$. Find equations for L_1 in the form
$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

2. Let L_2 be the line $x = y = z$. Find the shortest distance from L_1 to L_2.

3. Find the point of intersection of L_2 and P_1.

4. Find the equations of the line parallel to L_2 which passes through $(10, 1, -2)$.

5. Find the shortest distance from the point $(5, 1, 3)$ to P_1.

6. Find the shortest distance from the point $(5, 1, 3)$ to L_1.

7. Does there exist a matrix $T : \mathbb{R}^3 \to \mathbb{R}^3$ with
$$T \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad \text{and} \quad T \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}?$$
If so, find the matrix of such a T.

8. Let $f : \mathbb{R}^m \to \mathbb{R}^n$ be a linear mapping with matrix T. Which, if any, of the following statements are logically equivalent to each other? (They are logically equivalent if they mean exactly the same thing.)
   (a) T has a right inverse.
   (b) T has a left inverse.
   (c) T is invertible.
   (d) f is one-to-one.
   (e) f is onto.
   (f) f is one-to-one and onto.

9. Can you find a right inverse to the matrix $T = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \end{pmatrix}$? What about a left inverse? Is T the matrix of a one-to-one mapping? An onto mapping?