Math 8245 **Group Theory** **Fall Semester 2018**

Instructor

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Office Hours: MWF 11:15 - 12:50 or by appointment.

Course Content and Goals

The overall goal of the first semester is to teach a range of topics which go beyond the basic algebra course Math 8201/2 and which should be known by, and be useful to, the well-educated practitioner of group theory. I intend everything we study either to be interesting in its own right in some reasonable sense, or else a result we need to get to something else.

For the first 10 weeks or so of this semester there will be two parallel courses, one addressing the theoretical side of finite groups, the other dealing with computational group theory using the computer system GAP (computational methods being essential these days). The theoretical side will be taught on Mondays and Fridays each week, and on Wednesdays we will meet in Lind Hall 40, which is a computer classroom. No prior programming experience is necessary. We will start by learning the basics of the language that GAP uses, and go on to learn how to do computations with groups given as permutation groups, by presentations, and as matrix groups. As part of this we will learn how the algorithms we use work, and thereby gain some insight into their limitations. GAP is also available in SAGE, but in order to use any except the most rudimentary commands you need to know what they are in GAP.

On the theoretical side we will start with some basics of group theory: semidirect products, wreath products, Sylow subgroups of symmetric groups, actions on sets, semidihedral, dihedral and quaternion groups, groups of order the cube of a prime. We will then consider some more specialized things: stabilizer chain algorithms.; the Todd-Coxeter algorithm for coset enumeration, uses of Sylow's theorem beyond Math 8201, solvable and nilpotent groups. We will then go on to Bass-Serre theory (groups acting on trees and the associated free decompositions of the groups). This theory tells you everything you might want to know about the group SL(2,Z), for instance. If there is time in the first semester we will then do an introduction to finite simple groups, including construction of the Mathieu groups and simplicity of PSL groups.

Where we go after that in the second semester will be influenced by class interest. The topics I consider are Coxeter groups, Garside theory, groups with a BN-pair, group cohomology, crystallographic groups, aspects of representation theory.

Texts and sources

There will be no text to purchase. I will distribute handouts in class describing the theory we are covering and indicate where it can be found in books. We will study GAP using materials which you can download from my home page. I list some books below which will probably be useful.

Books on theoretical aspects

DJS Robinson, A course in the theory of groups, 2nd edition, Springer-Verlag, ISBN 0387944613

J. Rotman, An introduction to the theory of groups.

M. Isaacs, Finite Group Theory, AMS 2008, QA177 I835 2008.

O. Bogopolsky, Introduction to group theory, European Math. Soc. 2008, QA174.2.B64 2008 (general group theory, Mathieu groups, trees)
W. Dicks and M. Dunwoody, Groups acting on graphs, Cambridge U.P. (more information than we need about trees).
D.L. Johnson, Presentations of groups, Cambridge University Press 1990, ISBN 0521378249, chapters 8 and 9 (useful for presentations and coset enumeration).

Sources for GAP and computational group theory
D.F. Holt, B. Eick, E.A. O'Brien, Handbook of computational group theory, Chapman & Hall/CRC, c2005 (online access from the library eISBN 978-1-4200-3521-6)
The book by Johnson listed above is good for coset enumeration.
Information about GAP is obtained from its web site http://www.gap-system.org/
From the GAP site you can download GAP free of charge to your own computer. You may wish to explore the site, clicking on 'teaching', 'learning' and 'examples', for instance.
C.C. Sims, Computation with finitely presented groups, Cambridge University Press 1994, ISBN 0521432138

Course Assessment
I will assign a set of homework problems roughly every 2 weeks, giving a total of six homework assignments altogether. If you make a genuine attempt at 50% or more of the questions you will get an A for the course. You do not have to obtain correct solutions to these questions, only make genuine attempts (in my opinion). I believe that it is extremely difficult to obtain a sound and permanently lasting command of the material presented without doing some work which actively involves the student. It should be possible for everyone who wishes to obtain an A on this course. We can discuss whether some classes should be problem sessions, etc.

Expectations of written work
Give explanations for the calculations and arguments you make, written out in sentences i.e. with verbs, capital letters at the beginning, periods at the end, etc. and not in an abbreviated form.
Some of the homework will be computer exercises in GAP. Hand in a transcript of a GAP session. It will help if you insert explanatory comments. Please give me a hard copy of what you have done.
I encourage you to form study groups. However everything to be handed in must be written up in your own words. If two students hand in identical assignments, they will both receive no credit.

Prerequisites
The content of the Math 8200 algebra sequence is sufficient as a prerequisite. As far as group theory is concerned, the topics I expect you to know include:
Lagrange's theorem, the isomorphism theorems, direct products of groups, properties of permutations (they may be written as products of disjoint cycles, conjugacy classes in the symmetric groups, the sign), structure of finitely-generated abelian groups, Jordan-Hölder theorem, Sylow's theorems, basic properties of solvable groups, especially properties of the derived subgroup, the order of a finite general linear group.

Date of this version of the schedule: 8/31/2018