Preliminaries and Objectives

Preliminaries:
- Graphs of $\sin x$ and $\cos x$
- Amplitude
- Period (Wavelength)

Objectives:
- Draw an accurate graph of $y = A \sin(Bx + C) + D$ and $y = A \cos(Bx + C) + D$ over several periods.
$y = 2 \sin x + 3$
$y = 2 \sin x + 3$

Diagram showing the graph of $y = 2 \sin x$ and $y = \sin x$ with key points labeled: $\frac{\pi}{2}$, $\pi$, $\frac{3\pi}{2}$, and $2\pi$. The graph of $y = 2 \sin x$ is shifted upwards by 3 units compared to $y = \sin x$. The title reads: "University of Minnesota Phase Shift".
\[ y = 2 \sin x + 3 \]
General Forms of Wave Equations

\[ y = A \sin(Bx + C) + D \]

\[ y = A \cos(Bx + C) + D \]
$y = \sin(4x + \frac{\pi}{4})$
$y = \sin(4x + \frac{\pi}{4})$
\[ y = \sin(4x + \frac{\pi}{4}) \]
Phase Shift:

\[ 4x + \frac{\pi}{4} = 0 \]

\[ \Rightarrow 4x = -\frac{\pi}{4} \]

\[ \Rightarrow x = -\frac{\pi}{16} \]
$y = \sin(4x + \frac{\pi}{4})$
\[ y = \sin(4x + \frac{\pi}{4}) \]

Period \[ = \frac{2\pi}{4} = \frac{\pi}{2} \]
\[ y = \sin(4x + \frac{\pi}{4}) \]

\[
\begin{align*}
\text{Period} &= \frac{2\pi}{4} = \frac{\pi}{2} \\
Q &= \frac{\pi}{8}
\end{align*}
\]
\( y = \sin(4x + \frac{\pi}{4}) \)

\[ y = \sin(4x) \]

Period \( = \frac{2\pi}{4} = \frac{\pi}{2} \)

\( Q = \frac{\pi}{8} \)

Phase Shift \( = -\frac{\pi}{16} \)
$y = \sin(4x + \frac{\pi}{4})$

Period $= \frac{2\pi}{4} = \frac{\pi}{2}$

$Q = \frac{\pi}{8}$

Phase Shift $= -\frac{\pi}{16}$
\[ y = \sin(4x + \frac{\pi}{4}) \]

\[ y = \sin(4x) \]

\[ y = \sin(4x + \frac{\pi}{4}) \]

\[ y = \sin(4x) \]

\[ \text{Period} = \frac{2\pi}{4} = \frac{\pi}{2} \quad Q = \frac{\pi}{8} \quad \text{Phase Shift} = -\frac{\pi}{16} \]
General Procedures

Marking the $y$-axis:

- Find the amplitude
- Find the vertical shift
- Mark the $y$-axis with the vertical shift as the center line, and from the centerline, go above and below by the amplitude
General Procedures

Marking the $x$-axis:
- Find the period
- Find the quarter marks
- Find the phase shift
- Find a common denominator for the quarter marks and phase shift
- Mark the $x$-axis

Drawing the Graph:
- Mark the reference points (top, middle and bottom of wave) for the unshifted graph and draw the graph.
- Mark the reference points (top, middle and bottom of wave) for the shifted graph and draw the graph.
\[ y = -6 \cos\left(2x - \frac{\pi}{3}\right) + 2 \]

- Amplitude = 6
\[ y = -6 \cos\left(2x - \frac{\pi}{3}\right) + 2 \]

- Amplitude = 6
- Vertical Shift = 2
$y = -6 \cos\left(2x - \frac{\pi}{3}\right) + 2$

- Amplitude = 6
- Vertical Shift = 2
- Centerline = 2, Top = 8, Bottom = -4
\[ y = -6 \cos\left(2x - \frac{\pi}{3}\right) + 2 \]

- Amplitude = 6
- Vertical Shift = 2
- Centerline = 2, Top = 8, Bottom = -4
- Period = \( \pi \)
$y = -6 \cos(2x - \frac{\pi}{3}) + 2$

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- Period = $\pi$
- Quarter marks every $\frac{\pi}{4}$
\[ y = -6 \cos(2x - \frac{\pi}{3}) + 2 \]

- Amplitude = 6
- Vertical Shift = 2
- Centerline = 2, Top = 8, Bottom = -4
- Period = \( \pi \)
- Quarter marks every \( \frac{\pi}{4} \)
- Phase Shift: \( 2x - \frac{\pi}{3} = 0 \Rightarrow x = \frac{\pi}{6} \)
$y = -6 \cos(2x - \frac{\pi}{3}) + 2$

- Amplitude = 6
- Vertical Shift = 2
- Centerline = 2, Top = 8, Bottom = -4
- Period = $\pi$
- Quarter marks every $\frac{\pi}{4}$
- Phase Shift: $2x - \frac{\pi}{3} = 0 \Rightarrow x = \frac{\pi}{6}$
- $Q = \frac{3\pi}{12}$ \hspace{1cm} Phase Shift = $\frac{2\pi}{12}$
\[ y = -6 \cos\left(2x - \frac{\pi}{3}\right) + 2 \]
$y = -6 \cos(2x - \frac{\pi}{3}) + 2$
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$y = -6 \cos(2x + \frac{\pi}{3}) + 2$

$y = -6 \cos(2x) + 2$
Given the graph, find the equation
Given the graph, find the equation

\[ y = A \cos(Bx + C) + D \]
Given the graph, find the equation

\[ y = A \cos(Bx + C) - 1 \]
Given the graph, find the equation

\[ y = 2 \cos(Bx + C) - 1 \]
Given the graph, find the equation

\[ y = 2 \cos(3x + C) - 1 \]
Given the graph, find the equation

\[ y = 2 \cos(3x - \frac{\pi}{4}) - 1 \]
Given the graph, find the equation

$$y = A \sin(Bx + C) + D$$
Given the graph, find the equation

$$y = 2 \sin(3x + C) - 1$$
Given the graph, find the equation

\[ y = 2 \sin(3x + \frac{\pi}{4}) - 1 \]
Given the graph, find the equation

\[ y = -A \cos(Bx + C) + D \]
Given the graph, find the equation

\[ y = -2 \cos(3x + C) - 1 \]
Given the graph, find the equation

\[ y = -2 \cos(3x + \frac{3\pi}{4}) - 1 \]
Recap

\[ y = A \cos(Bx + C) + D \]

- \( D \) determines the \( y \)-value of the centerline
- \( A \) = amplitude, which is added/subtracted from the centerline value to determine the tops and bottoms of the waves.
- Period = \( \frac{2\pi}{B} \), divide by 4 to place quarter markings
- Set \((Bx + C) = 0\) to find phase shift. Mark the \( x \)-axis by finding a common denominator between the phase shift and the quarter markings.
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