Domain and Range of Trig and Inverse Trig Functions

UNIVERSITY OF MINNESOTA

Preliminaries and Objectives

Preliminaries:

- Graphs of $y = \sin x$, $y = \cos x$ and $y = \tan x$.

Objectives:

- Find the domain and range of basic trig and inverse trig functions.

Domain and Range of General Functions

- The **domain** of a function is the list of all possible inputs (*x*-values) to the function.

- The **range** of a function is the list of all possible outputs (*y*-values) of the function.

- Graphically speaking, the domain is the portion of the *x*-axis on which the graph casts a shadow.

- Graphically speaking, the range is the portion of the *y*-axis on which the graph casts a shadow.

Domain and Range

Function	Domain	Range
$y = sin(x)$		
$y = cos(x)$		
$y = tan(x)$		
$y = sin^{-1}(x)$		
$y = cos^{-1}(x)$		
$y = tan^{-1}(x)$		

Domain and Range

Function	Domain	Range
$y = sin(x)$	$-\infty < x < \infty$	
$y = cos(x)$	$-\infty < x < \infty$	
$y = tan(x)$		
$y = sin^{-1}(x)$		
$y = cos^{-1}(x)$		
$y = tan^{-1}(x)$		

Domain and Range

Function	Domain	Range
$y = \sin(x)$	$-\infty < x < \infty$	$-1 \leq y \leq 1$
$y = \cos(x)$	$-\infty < x < \infty$	
$y = \tan(x)$		
$y = \sin^{-1}(x)$		
$y = \cos^{-1}(x)$		
$y = \tan^{-1}(x)$		

Domain and Range

Function	*Domain*	*Range*
$y = sin(x)$	$-\infty < x < \infty$	$-1 \leq y \leq 1$
$y = cos(x)$	$-\infty < x < \infty$	$-1 \leq y \leq 1$
$y = tan(x)$		
$y = sin^{-1}(x)$		
$y = cos^{-1}(x)$		
$y = tan^{-1}(x)$		

Domain and Range

Function	Domain	Range
$y = sin(x)$	$-\infty < x < \infty$	$-1 \le y \le 1$
$y = cos(x)$	$-\infty < x < \infty$	$-1 \le y \le 1$
$y = tan(x)$	$x \ne \ldots -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots$	
$y = sin^{-1}(x)$		
$y = cos^{-1}(x)$		
$y = tan^{-1}(x)$		

Domain and Range

Function	*Domain*	*Range*
$y = \sin(x)$	$-\infty < x < \infty$	$-1 \leq y \leq 1$
$y = \cos(x)$	$-\infty < x < \infty$	$-1 \leq y \leq 1$
$y = \tan(x)$	$x \neq \ldots - \frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots$	$-\infty < y < \infty$
$y = \sin^{-1}(x)$		
$y = \cos^{-1}(x)$		
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Domain and Range

Function	Domain	Range
$y = sin(x)$	$-\infty < x < \infty$	$-1 \leq y \leq 1$
$y = cos(x)$	$-\infty < x < \infty$	$-1 \leq y \leq 1$
$y = tan(x)$	$x \neq \ldots -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots$	$-\infty < y < \infty$
$y = sin^{-1}(x)$	$-1 \leq x \leq 1$	
$y = cos^{-1}(x)$	$-1 \leq x \leq 1$	
$y = tan^{-1}(x)$		

Domain and Range

Function	*Domain*	*Range*
$y = \sin(x)$	$-\infty < x < \infty$	$-1 \leq y \leq 1$
$y = \cos(x)$	$-\infty < x < \infty$	$-1 \leq y \leq 1$
$y = \tan(x)$	$x \neq \ldots -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots$	$-\infty < y < \infty$
$y = \sin^{-1}(x)$	$-1 \leq x \leq 1$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
$y = \cos^{-1}(x)$	$-1 \leq x \leq 1$	
$y = \tan^{-1}(x)$		

Domain and Range

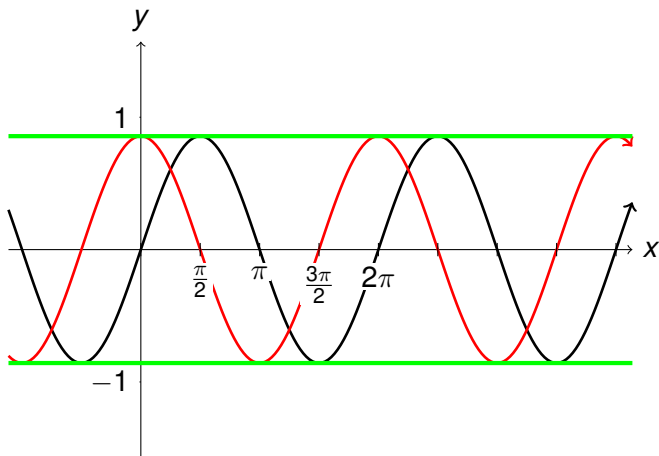| Function	Domain	Range
$y = sin(x)$	$-\infty < x < \infty$	$-1 \leq y \leq 1$
$y = cos(x)$	$-\infty < x < \infty$	$-1 \leq y \leq 1$
$y = tan(x)$	$x \neq \ldots - \frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots$	$-\infty < y < \infty$
$y = sin^{-1}(x)$	$-1 \leq x \leq 1$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
$y = cos^{-1}(x)$	$-1 \leq x \leq 1$	$0 \leq y \leq \pi$
$y = tan^{-1}(x)$		

Domain and Range

Function	Domain	Range
$y = \sin(x)$	$-\infty < x < \infty$	$-1 \leq y \leq 1$
$y = \cos(x)$	$-\infty < x < \infty$	$-1 \leq y \leq 1$
$y = \tan(x)$	$x \neq \ldots -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots$	$-\infty < y < \infty$
$y = \sin^{-1}(x)$	$-1 \leq x \leq 1$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
$y = \cos^{-1}(x)$	$-1 \leq x \leq 1$	$0 \leq y \leq \pi$
$y = \tan^{-1}(x)$	$-\infty < x < \infty$	

Domain and Range

Function	*Domain*	*Range*
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Domain, Range and Graphs

$$y = \sin x \qquad y = \cos x$$

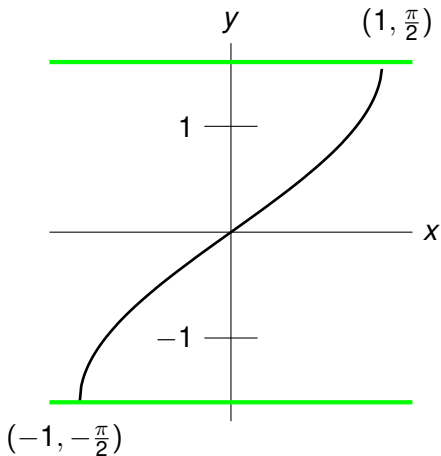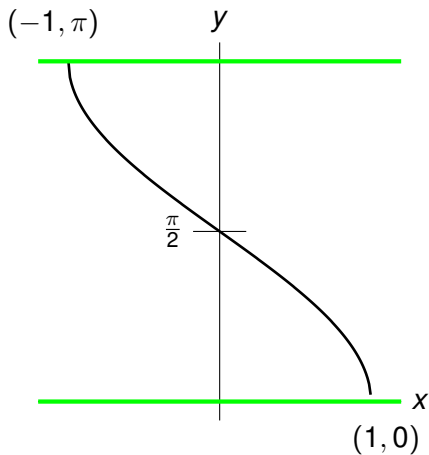# Domain, Range and Graphs

$$y = \tan x$$

Domain, Range and Graphs

$$y = \sin^{-1} x$$
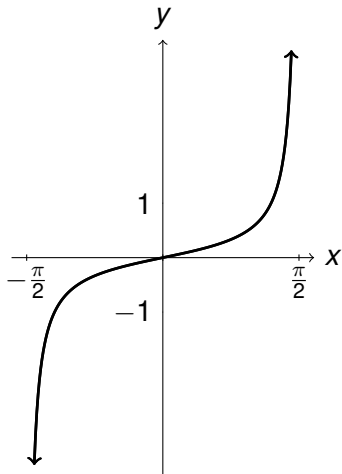
$$y = \cos^{-1} x$$

Domain, Range and Graphs

$$y = \tan^{-1} x$$

Recap

Function	Domain	Range
$y = sin(x)$	$-\infty < x < \infty$	$-1 \leq y \leq 1$
$y = cos(x)$	$-\infty < x < \infty$	$-1 \leq y \leq 1$
$y = tan(x)$	$x \neq \ldots - \frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots$	$-\infty < y < \infty$
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$y = tan^{-1}(x)$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

Credits

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