

1. Values of Tangent and Inverse Tangent

2. You should be familiar with the unit circle values for sine and cosine, and with the function inverse sine and inverse cosine.

In this lesson, we will find unit circle values for the tangent function and inverse tangent function.

3. We have found values for sine and cosine for some special angles. Here is a table of values in the first quadrant with the angles measured in degrees.

4. We can also measure the angles in radians. We can find the value of tangent for these angles by using the definition tangent equals sine divided by cosine. To find  $\tan 0$ , we divide  $\sin 0 / \cos 0$  that is, we divided  $0/1$ , and get  $0$ . To find  $\tan \frac{\pi}{6}$ , we divide  $\sin \frac{\pi}{6} / \cos \frac{\pi}{6}$  that is, we divided  $\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$ . The twos cancel, so we get  $\frac{1}{\sqrt{3}}$  which we can simplify to  $\frac{\sqrt{3}}{3}$ . The tangent of  $\frac{\pi}{4}$  is  $1$ . The tangent of  $\frac{\pi}{3}$  is  $\sqrt{3}$ . The tangent of  $\frac{\pi}{2}$  is  $1/0$ , which is undefined.

5. Having established the first quadrant values, we can then use the symmetry of the circle to find values in the second quadrant,

6. the third quadrant and the fourth quadrant. Notice that the values of tangent are positive in the third quadrant, since both the sine and cosine are negative. The values of tangent are negative in the second and fourth quadrants.

7. We can also go clockwise around the circle to get values for tangent for negative angles.

To find angles for the inverse tangent function, we must make a choice similar to the choice we made for sine and cosine. It is natural to use the first quadrant for the positive values. The tangent is undefined at  $\frac{\pi}{2}$  and  $-\frac{\pi}{2}$ , so it makes sense to use those two angles as the boundaries. We pick the negative angles in the fourth quadrant to be the answers to inverse tangent. You can read this table in reverse to find inverse tangent values. For example, the inverse tangent of  $-\sqrt{3}$  is  $-\frac{\pi}{3}$ , since  $-\frac{\pi}{3}$  is the angle whose tangent is  $-\sqrt{3}$ .

8. We can also write out the angles that satisfy tangent values using the inverse tangent notation. Here are 7 values for which you should know the inverse tangent.

9. To recap: Values for the tangent function can be found by dividing sine by cosine. Values for the inverse tangent function will be found in the first quadrant for positive inputs and in the fourth quadrant, measured as negative angles for negative inputs.