1. General Equation of a Hyperbola

2. You should be familiar with transformations of graphs

   In this lesson, we will graph hyperbolas, and write the equation of a hyperbola, given its graph.

3. Recall that we arrived the general equation of an ellipse by stretching the unit circle $x^2+y^2=1$

4. The equation of the standard hyperbola is similar to the equation for a circle or an ellipse, except one of the squared terms is negative. To see what the graph looks like, we first examine the intercepts. If $x=0$, then we are left with the equation $-y^2=1$ which has no solutions, so this graph will not touch the y-axis. If $y=0$, we get the equation $x^2=1$, so we will have intercepts at -1 and 1.

5. The points $(1,0)$ and $(-1,0)$ are called the **vertices**. As we move up and to the right, the graph will approach the line $y=x$. We will demonstrate this behavior with an example. If $x=10$, then $x^2=100$ so $y^2=99$ and $y=\sqrt{99}$, which is just slightly less than 10. The further we go to the right, the closer the y value is to the x value.

6. To draw the graph, we need to draw these lines that the graph approaches, called asymptotes. For the standard hyperbola, we draw a box whose corners are up and down one, and left and right one.

7. We then draw the asymptotes diagonally through the box.

8. and sketch the graph from the vertices toward the asymptotes.

9. For the hyperbola $y^2-x^2=1$, we have the same asymptotes, but the vertices are now on the y-axis at ± 1.

10. We can stretch the hyperbola the same way we stretched the ellipse. Dividing x^2 by a^2 stretches the graph left and right by a factor of a. Similarly, dividing y^2 by b^2 stretches the graph up and down by a factor of b.

11. Here is an example. The x^2 term is positive and the y^2 term is negative, so this is a hyperbola with vertices on the x-axis. The horizontal stretch factor is 2, so the vertices have x values of ± 2. The vertical stretch factor is 3, so the asymptotes go through points which are 2 units left and right and 3 units up and down. The hyperbola is then graphed, starting at the vertex, and heading left and right toward the asymptotes. If we wish, we could shift the graph as we have done with other graphs.

12. Here is an example in the other direction. We are given the graph, and wish to write the equation. The hyperbola is headed left and right from the vertices, so this is a hyperbola whose x term is positive.

13. The center of the graph is at a height of -1. The x-value of the center is halfway between -1 and 5, which is 2, so the center is at $(2,-1)$. We subtract these values inside the parentheses.

14. The vertices are a distance of 3 left and right from the center, so the horizontal stretch factor is 3. The asymptote box has corners that are 5 above and below the vertices, so the vertical stretch factor is 5.

15. The squares of the stretch factors appear in the denominators.

16. To recap: If the x^2 term is positive, the vertices will be on the x-axis, a distance of a to the left and right of center. The asymptotes are drawn diagonally through a box whose corners are a distance of a left and right of center, and a distance of b above and below.

17. If the y^2 term is positive, we follow the same procedure, except the hyperbola goes up and down instead of left and right. The vertices are a distance of b above and below the center.